The University of Chicago

HOW NUMERALS ARE READ

AN EXPERIMENTAL STUDY OF THE READING OF ISOLATED NUMERALS AND NUMERALS IN ARITHMETIC PROBLEMS

A DISSERTATION

SUBMITTED TO THE FACULTY
OF THE GRADUATE SCHOOL OF ARTS AND LITERATURE
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF EDUCATION

BY

PAUL WASHINGTON TERRY

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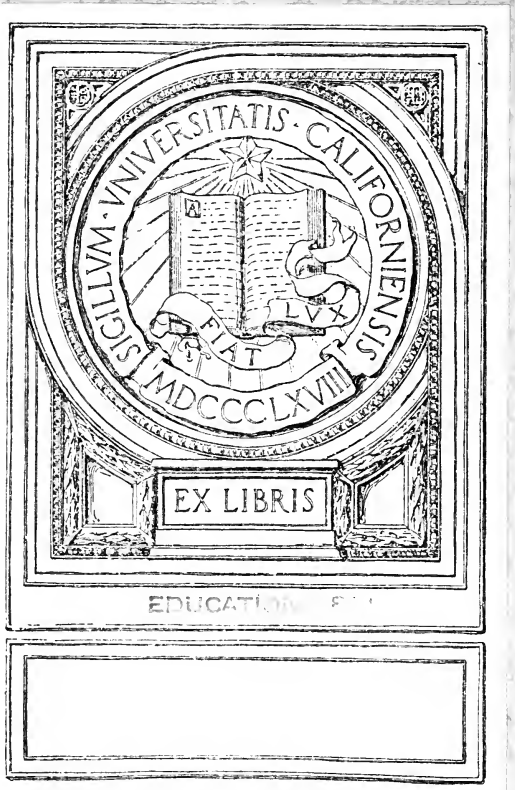
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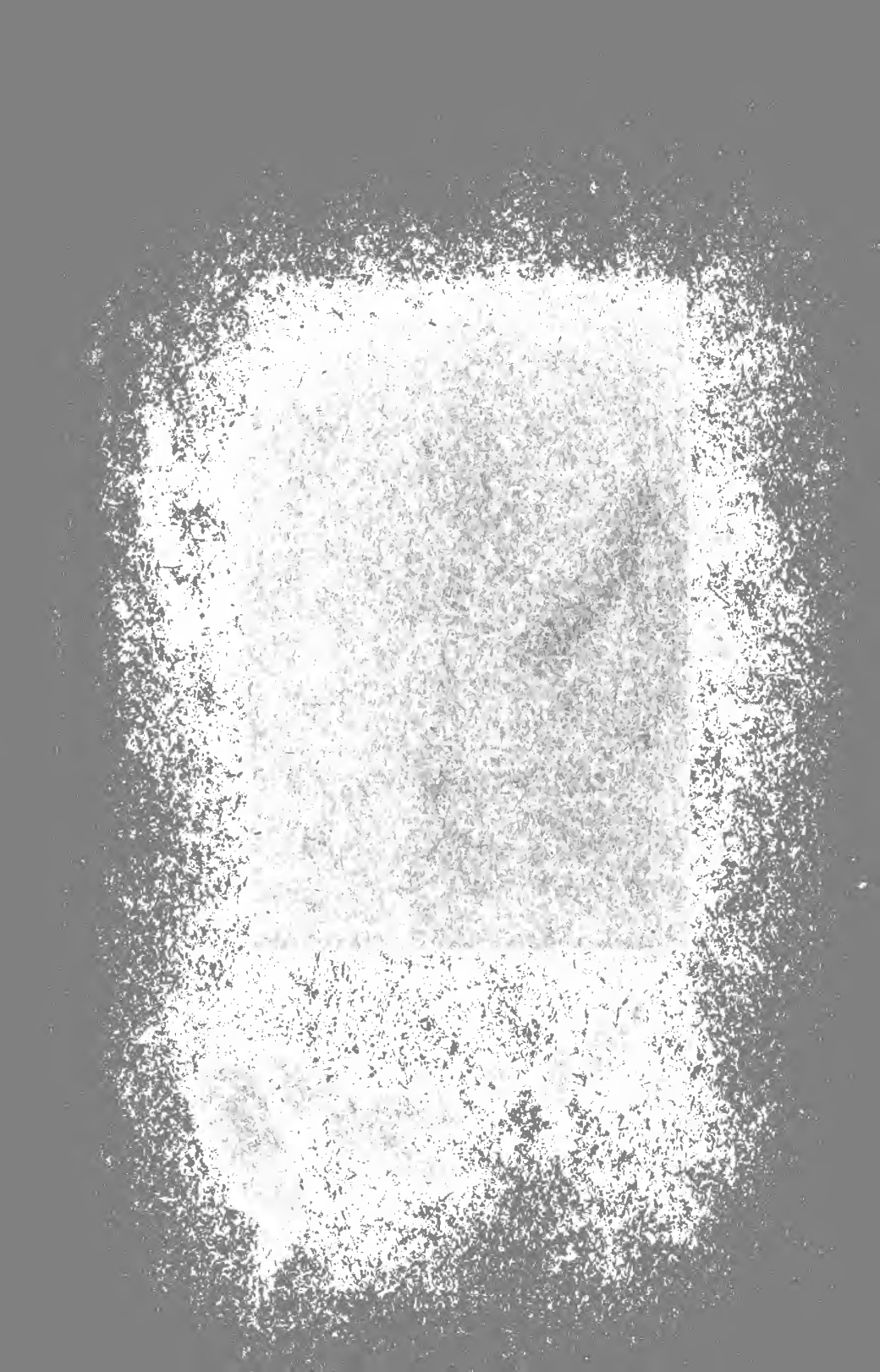

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CHAPTER I

INTRODUCTION

Numerous investigations in the psychology of reading and in the measurement of reading ability have developed a valuable body of scientific information about the methods of reading words and sentences of the ordinary kind, but the reading of numerals has had only occasional attention in these studies and that merely in an incidental way. Such work on reading as has been done in the field of arithmetic has concerned itself with isolated numerals rather than with numerals set in sentences or problems.

The present investigation is concerned with the reading of numerals both in separate lines and in the context of arithmetical problems. The first part of this report describes a series of studies, based on introspective observations, of some of the relatively definite and highly developed habits of graduate students in reading numerals both isolated and in problems. The second part of the report deals with this same class of readers and with the same kinds of reading materials, but employs objective methods.

For the first part of the report the data were obtained by recording introspective observations which were made by the subjects after they had read a set of arithmetical problems in which numerals occurred. The introspections were supplemented by directly observing and reporting the results of the reading of isolated numerals. The information secured in this preliminary work serves as a basis for the interpretation of the data obtained in the second part of the investigation in which photographic records of eye-movements were secured. The whole investigation is only an introduction to the study of the methods employed by children in their gradual acquisition of the power of reading numerals. This large genetic study was the original aim of the present investigation. The intricacies of the problem turned what was originally thought of as an introductory investigation into an elaborate detailed study. Yet educational implications are present even in this preliminary work. Through the study of adults, a body of facts has been discovered which throws light on methods of reading problems in arithmetic to which children must ultimately attain, whatever be the initial habits through which they pass in the course of their development.

PART I. PRELIMINARY STUDIES OF THE READING OF NUMERALS—BY INTROSPECTIVE METHODS

CHAPTER II

NUMERALS IN ARITHMETICAL PROBLEMS—FIRST PRELIMINARY STUDY

1. DESCRIPTION OF THE STUDY

For the first preliminary study seven simple arithmetical problems were used. These were so formulated that each included a set of from one to four numerals. The problems were so made up that while the numerals in each one were similar, those which were used in the different problems exhibited variations in length. The numerals in problems 1, 3, 5, and 6 are two in number in each case, but vary in digit-length from one to seven digits. Problem 2 includes a set of four numerals, each numeral being made up of from one to two digits; Problem 4 has four numerals made up of from three to four digits; and Problem 7 uses a familiar date and two numerals of exceptional character, namely 100 and 1000.

The problems which were used in the first preliminary study are as follows:

1. At 65 cents a dozen, what will 8 dozen eggs cost?

2. A man buys 5 tons of coal at 9 dollars a ton, and 3 cords of wood at 12 dollars a cord. What is the total cost of both of them?

3. A farmer owns one farm of 286 acres, and another of 1754 acres. How many acres does he own all together?

4. A wholesale grocer has 4375 cases of canned corn. To three customers he shipped 286, 2567, and 615 cases respectively. How many did he have left?

5. If electricity travels on a wire at the rate of 288,106 miles per second, how long will it take to travel 144,053 miles?

6. If one railroad uses 2,191,504 cross ties during the year, and another railroad 617,450 in the same period of time, how many more ties does the one use than the other?

7. During 1918 a citizen bought four $100 Liberty bonds, and two $1000 bonds. What is the total of the sum he invested in these bonds?

Ten graduate students of the School of Education of the University of Chicago were asked to solve all of the problems. They were instructed

to work the problems rapidly and accurately, and with pencil and paper or without, as they preferred. They were urged to observe faithfully the arithmetic problem-solving attitude from the beginning of a problem to its solution. After the answer of each problem was recorded the subjects were asked to describe in detail their experiences while reading the problem with special reference to the numerals. After the first problem had been solved and the reading of its numerals described, the subjects began to note the kinds of experiences they were asked to observe. As a result they were able to give the desired description more promptly and easily with each successive problem. These were recorded and condensed into tables I–VI.

2. FIRST READING AND RE-READING DISTINGUISHED

The most obvious and general fact noted in the records of the several subjects was their clear and unmistakable differentiation of the reading of a problem into two definite and distinct phases differing in time and in purpose, namely, the first reading and the re-reading. Subsequent sections of the investigation emphasize the importance of this observation concerning the distinction between two phases of the reading of a problem. The general procedure of each subject was, first, to read the problem through "to get the sense" or "to see what was to be done with the numbers," and secondly, to re-read one or all of the numerals, and sometimes also a few of the accompanying words. These re-readings of the numerals were for such purposes as "verification" of their first reading, or the "cultivation of assurance" before copying the figures on paper for computation. The subject with one or two exceptions was not aware that he habitually followed such a procedure until he began to make introspective observations of his habits.

3. PARTIAL FIRST READING AND WHOLE FIRST READING OF NUMERALS

The knowledge gained during the first reading was found to be very different in different cases. Subjects sometimes perceived numerals as merely numerals; sometimes they noted only the first digit or the first two digits. At times they noted the number of digits but did not attend to any one in particular. Again they reported a numeral as large or as small, or as larger or smaller than some other numeral. Sometimes they noted its location in the typewritten line. Frequently two or more of these items were included in the general perception. In all such cases as have been described, the perception lacks detail and precision. It is evidently a kind of cursory preliminary recognition of the

general character and setting of the numeral. Its value consists in the fact that it permits the subject to think about the problem without entering at first into the minute details of solution.

There were cases, however, even in the first readings, in which the subjects gave attention to the identity and place of every component digit. In addition these careful readers noted also the character of the numeral, observing whether it was a whole number or a decimal. They also gained a notion of the magnitude of the numeral as determined by the number of digits. With each of the subjects, cases were found in which the recognition was full and detailed. Such cases were recorded as "whole first readings." Any reading which fell short of complete detail was recorded as a "partial first reading."

The results of the introspections are given in full in Table I. Beginning at the top of the left-hand column, this table should be read vertically as follows: Problem 1 contains the two numerals 65 and 8, and these

TABLE I

PARTIAL FIRST READINGS AND WHOLE FIRST READINGS BY TEN SUBJECTS IN SEVEN PROBLEMS

	Problems							
	1	2	3	4	5	6	7	
Numerals given in problems	65 8	5 9 3 12	386 1754	4375 286 2567 615	288,106 144,053	2,191,504 617,450	1918	100 1000
Total number of readings	20	40	20	40	20	20	10	20
Partial first readings	3	12	10	31	13	11	0	1
Whole first readings	16	28	10	9	7	9	10	19
Doubtful	1							

were read altogether a total of twenty times by the ten subjects. Of the twenty readings, three were partial first readings, sixteen were whole first readings, and one could not be classified.

Examination of Table I shows that the numerals in problems 4, 5, and 6, which were all longer ones with three to seven digits, were read partially more than half of the times and accordingly have percentages of partial first readings of 50 or more. The numerals in problems 1, 2, and 7, on the other hand, were read wholly more than half of the times and have percentages of partial first readings of only 30 or less. The longer numerals are seen to have been read partially more frequently, while the shorter numerals are read in detail more frequently. Attention should be called at this point, however, to the fact that the whole first reading of a numeral does not necessarily mean that the numeral will

not be re-read. On the contrary later discussions in this report will show that almost all numerals were re-read after the first reading, including even those which were read in detail during the first reading.

In order to bring out the relation between partial and whole readings and the character of the numerals, Table II was compiled. This table shows the ranks of the various numerals with reference to the frequency of partial readings. Percentages were calculated by dividing the number of times a numeral was partially read by the total number of readings of that numeral.

Among the longer numerals a greater digit-length appears to cause a large percentage of partial readings. Such a comparison between numerals of different lengths is significant when the same number of numerals is used in the various problems compared. Problems 3, 5,

TABLE II

RANKS OF NUMERALS ACCORDING TO PERCENTAGES OF PARTIAL FIRST READINGS

Ranks	Percentage of Partial First Readings	Description of Numerals	Numerals Read
1	77	Four three- to four-digit	4375; 286; 2567; 615
2	65	Two six-digit	288,106; 144,053
3	55	Two six- to seven-digit	2,191,504; 617,450
4	50	Two three- to four-digit	386; 1754
5	30	Four one- to two-digit	5; 9; 3; 12
6	15	Two one- to two-digit	65; 8
7	5	Two familiar	100; 1000
8	0	Date	1918

and 6 each have two numerals. The six digit numerals of Problem 5, and the six- and seven-digit numerals of Problem 6 were given respectively 65 per cent and 55 per cent of partial first readings. The three- and four-digit numerals of Problem 3, on the other hand, were given only 50 per cent of partial first readings.

Of the numerals which were usually given a whole first reading, the date 1918 stands out as different in character from other numerals of like length. It is the only one which was never partially read. The very familiar numerals 100 and 1000 with the dollar sign attached were like the date for the most part, in that they were read partially only once. In this case, the partial reading was revealed by a mistake made by the reader—the numeral 100 was read as 1.00 instead of as 100.

The inclusion of several numerals in the same problem appears to induce a greater proportion of partial first readings. In Problem 4,

where there are four numerals, the percentage of partial readings is 77, whereas in Problem 3, where only two numerals appear, the percentage of partial readings is only 50, although the numerals in both problems are of the same lengths. The explanation of this fact seems to be that the subject loses interest in the numerals when many of them appear together. Consequently he does not make the radical adjustments in rate of reading which would be necessary for the careful reading of a series of several numerals.

The validity of this explanation is supported by the results of another comparison of a similar type which can be made from the tables. The numerals in Problems 1 and 2 are all one or two digits in length. There are two numerals in Problem 1, and four in Problem 2. The percentage of partial readings in Problem 1 is 15, whereas in Problem 2 the percentage of partial readings is 30, or twice as great as that in Problem 1. In this comparison, as in that above, where four numerals of a certain digit-length appear in a problem, they were more frequently read partially than when only two such numerals appear.

The first numeral in a problem tends to be given a more careful and thorough reading than any of the other numerals in the same problem. The basis for this statement is found in the fact that in three of the four problems which employ the longer numerals, the first numeral receives a greater number of whole first readings than any of the numerals that follow. According to the original tabulations, the first numeral in Problem 5, 288,106, was given five whole first readings whereas the second numeral, 144,053, was given only two whole first readings. A similar preponderance of whole readings appears in favor of the first numeral in both problems 3 and 4. A comparison of the foregoing kind cannot be drawn, however, between shorter numerals, since they were almost invariably given whole first readings regardless of their position within the problem.

4. INDIVIDUAL SUBJECTS CLASSIFIED AS PARTIAL FIRST READERS AND AS WHOLE FIRST READERS

In Table III the ten subjects are arranged in order from left to right according to the number of their partial readings. They range from 14 partial first readings by G to no partial first readings by Subject H. The total in each case is 19 readings. G, Bl, and S show a significant preponderance of partial readings. H, T, D, and K, on the other hand, exhibit a preponderance of whole first readings, each of the latter showing 12 or more such readings out of a possible 19. These seven subjects

can accordingly be classified into two groups as partial first readers and whole first readers. The partial readers read partially not only the three- to seven-digit numerals usually thus read, but also several of the other numerals which are usually read in detail. Similarly the whole

TABLE III

SUBJECTS ARRANGED ACCORDING TO NUMBER OF PARTIAL AND WHOLE FIRST READINGS

	SUBJECTS									
	G	Bl	S	P	De	K	Ko	D	T	H
Partial first readings	14	13	12	9	8	7	7	6	3	0
Whole first readings	5	6	7	10	11	12	11	13	16	19
Doubtful	.....	.....	.....	.....	.....	.....	1	.....	.....	.....

readers read in detail not only the nine one- and two-digit numerals and the familiar numerals which are usually so read but also several of the other numerals which are usually read only partially.

Subjects Ko, De, and P are not so distinctly marked off as the seven discussed above. However, since they show a preponderant number of whole first readings, they may be classified as whole first readers. When they are so classified there are seven whole first readers and only three partial first readers. There were, therefore, more than twice as many whole first readers as partial first readers among the subjects of this study.

5. RE-READING OF THE SEVERAL NUMERALS

After the first reading of a problem it was left entirely to the choice of the subject whether he should or should not re-read the numerals in the problem. In all but a few cases, which are classified as "Doubtful," the reports of every subject show when he re-read any individual numeral. Table IV gives, for each set of numerals, the number of times they were

TABLE IV

NUMBER OF RE-READINGS

	Problems							
	1	2	3	4	5	6	7	
Numerals given in problems	65 8	5 9 3 12	386 1754	4375 286 2567 615	288,106 144,053	2,191,504 617,450	1918	100 1000 ....
Number of re-readings	11	38	20	39	17	20	0	13
Numerals not re-read	9	.....	.....	1	1	.....	10	7
Doubtful	.....	2	.....	.....	2	.....	.....	.....

re-read, the number of times they were not re-read, and the number of doubtful cases. Table V gives the ranks of the several sets of numerals according to the percentages of re-readings. The percentage of re-readings for any set of numerals was found by dividing the total number of re-readings which the numerals of the set received, by the total number of re-readings which it was possible for them to have received.

Examination of Table IV reveals the fact that the numerals of every set but one were very generally re-read. The longer numerals were re-read almost without exception. From Table V it is seen that the four sets of numerals from three to seven digits in length which are found in problems 3, 4, 5, and 6, received 85 per cent, 97.5 per cent, and 100 per cent, respectively, of the numbers of possible re-readings. In two cases only were numerals of this length reported as not re-read, and only

TABLE V

RANKS OF NUMERALS ACCORDING TO PERCENTAGES OF RE-READINGS

Ranks	Percentage of Re-readings	Description of Numerals	Numerals Read
1.5	100	Two six- to seven-digit	2,191,504; 617,450
1.5	100	Two three- to four-digit	386; 1754
3	97.5	Four three- to four-digit	4375; 286; 2567; 615
4	95	Four one- to two-digit	5; 9; 3; 12
5	85	Two six-digit	288,106; 144,053
6	65	Two familiar	100; 1000
7	55	Two one- to two-digit	65; 8
8	0	Date	1918

two cases were reported as doubtful. Since there are ten ordinary numerals of three- to seven-digit lengths, it was possible for these numerals to have been re-read a total of one hundred times by the ten subjects. Of this number of possible re-readings the three- to seven-digit numerals actually were re-read ninety-six times.

Even the short one- and two-digit numerals and the familiar numericals were very generally re-read, although usually they had been given whole first readings. Each of these sets of numerals, with the exception of the familiar date numeral, was re-read 50 per cent or more of the possible times. The one- and two-digit numerals, in problems which contain only two numerals, were re-read 55 per cent of the possible times; but the numerals of this same length, in problems which contain four numerals, were re-read 95 per cent of the possible times. The familiar numerals with the dollar sign attached were re-read 65 per cent of the possible times. The only numeral never re-read is the

familiar date 1918, which was not necessary in any way to the solution of the problem in which it appears.

6. RE-READING BY INDIVIDUAL SUBJECTS

The very large percentages of re-readings of the several sets of numerals imply that most of the individual subjects are persistent re-readers. Examination of Table VI, in which the facts for each individual subject are displayed, proves that such is the case. Subjects G, Bl, and S who had read most of the numerals only partially during the first reading, proceeded to re-read the numerals before they began to solve the problems. They re-read not only the numerals at which they had merely glanced the first time, but also those which they had read in detail at first reading. None of these three subjects showed a number of numerals not re-read equal to the number of numerals which he had read in detail at first reading.

TABLE VI

NUMBER OF RE-READINGS BY VARIOUS SUBJECTS

	SUBJECTS									
	G	Bl	S	P	De	K	Ko	D	T	H
Number of re-readings	16	18	16	13	17	17	18	16	12	18
Numerals not re-read	3	1	3	6	1	2	1	3	5	1
Doubtful	.....	.....	.....	.....	1	.....	.....	.....	2	.....

The seven subjects who were classified as whole first readers re-read practically as many of the numerals as the partial first readers. Subject H, who gave all of the numerals a whole first reading, re-read as many numerals as any partial first reader. The smallest numbers of numerals re-read are found in the records of the whole first readers, T and P. These same subjects have the largest numbers of numerals not re-read. Besides the familiar numerals and the one- and two-digit numerals, T did not re-read the numeral 4375 and P did not re-read 288,106.

At this time attention should be called to the fact, which will be elaborated in chapter viii, that the re-reading of numerals appears to be very closely connected with copying them on paper for computation. With rare exceptions the subjects, when solving the problems of this study, followed the procedure of copying the numerals and computing with pencil. In the chapter referred to above, however, it was found that several subjects, including G and H of the present study, solved the problems which were read in the second part of the investigation "men-

tally" and directly from the text. Such subjects when solving problems "mentally" did not re-read the numerals.

After each subject had solved all of the problems, he was asked to describe any individual attitudes or previous experiences which he believed had affected in an important way his methods of reading numerals in arithmetical problems. Three of the subjects gave descriptions which threw light upon their procedures as reported above. Subject T stated that he has had from his early school days unusual ability in retaining by visual memory both long and short numerals which he had read. This ability had recently undergone intensive training in the form of much reading and copying of army serial numbers, which his work as company clerk in the army required. He believes his habit is to read numerals in detail wherever he sees them, and that he could have solved perhaps all of the seven problems immediately after the first reading without looking at the numerals again. He goes on to say, however, that notwithstanding his careful first reading of the numerals, he is in the habit of re-reading them before beginning computation "in order to be sure."

Subject H, who gave every numeral a whole first reading, states that early difficulties with arithmetical problems caused him to develop habits of great caution in reading and solving them. He describes his procedure as beginning with a careful, complete reading of every numeral in a problem when he first comes to it. He then returns to re-read every numeral and sometimes some of the words.

Subject G, whose record presents the largest number of partial first readings of the numerals, reports that he intends to obtain "only a general idea" of the numerals from the first reading, especially if they are long ones. He explains that he chose this attitude toward the numerals after he had learned through experience that he was unable to recall them for computation and "had to go back for them anyhow."

7. SUMMARY

A summary of the results of the first preliminary study on how numerals in arithmetical problems are read includes the following points:

1. The subjects distinguished two phases in the reading of problems, namely, a first reading and a re-reading. The purpose of the first reading is to discover the conditions of the problem, while that of the re-reading is to perceive the numerals accurately for use in computation.

2. Two ranges of perception of numerals during the first reading are distinguished, namely, whole first reading and partial first reading.

3. Shorter numerals and very familiar numerals more frequently receive whole first readings, whereas longer numerals more frequently receive partial first readings.

4. The first numerals in problems which have numerals of three to seven digits in length, are commonly given whole first readings.

5. When as many as four numerals appear in a problem they receive a greater proportion of partial first readings than in those cases in which only two numerals of the same digit-length appear.

6. Subjects differ widely in their habits. Some are predominantly whole first readers, others are partial first readers.

7. Numerals of all lengths and types, when they are used in computation, are very generally re-read for computation.

8. All subjects persistently re-read numerals when they begin to use them in computation.

CHAPTER III

RANGE OF CORRECT RECALL OF NUMERALS AFTER THE FIRST READING—SECOND PRELIMINARY STUDY

1. DESCRIPTION OF THE STUDY

The purpose of the second preliminary study was to obtain further information as to the nature of the readings of the numerals during the first reading of a problem. The general plan followed for the accomplishment of this purpose was to have the subjects report all of the details of the numerals, which they were able to recall immediately after the first reading.

The subjects were seven graduate students in the School of Education of the University of Chicago. Of the seven subjects, only one, Subject G, had served in the first preliminary study. The materials which they read were twelve simple arithmetical problems similar to those used in the first preliminary study. Six of the problems were work problems in the sense that each of them was worked until an answer was found. No questions, however, were asked concerning the numerals of these problems. Their function was to help the subjects maintain throughout the study the problem-solving attitude by actually solving problems. The readings of the other six problems were the bases of the data which were used in the study. In the context of these problems a total of sixteen numerals varying in length from one to seven digits was included. Two or three numerals were placed in each of five of the problems. In Problem D, however, four numerals appeared together. Numerals of similar length were placed in the same problem, as was done in the problems of the first preliminary study. All of the problems were presented in typewriting.

The problems which were read for data are as follows:

PROBLEM A

At 43 cents a dozen, what will 2 dozen eggs cost?

PROBLEM B

If a man buys 5 tons of coal for 45 dollars, what will he have to pay for 8 tons?

PROBLEM C

A farmer owns one farm of 246 acres, and another of 1754 acres. How many acres does he own altogether?

PROBLEM D

A wholesale merchant had on hand 1000 cases of canned corn. From three factories he bought 1276, 91 and 718 cases respectively. How many did he then have?

PROBLEM E

If one railroad uses 2,981,534 cross ties during the year, and another railroad 617,453 in the same period of time, how many more ties does the one use than the other?

PROBLEM F

If one man can do a piece of work in 10 days, and another can do the same work in 9 days, what should be the wages of the second, if the wages of the first are 3 dollars?

The subjects were informed that they would be expected to solve most of the problems, but that they would be interrupted in their procedure in some of them by the placing of a cardboard screen over the text which was being read. They were told that they would not be informed beforehand as to which problems were to be solved and which were to be covered with the screen. Accordingly they were urged to maintain their normal problem-solving attitude toward every problem that was presented.

The time chosen for the placing of the screen, when it was to be used, was the moment when the subject completed the first reading of the problem. The subjects were therefore instructed to signal by a slight movement of the left hand the moment at which they were completing the first reading. This they were able to do without embarrassment after a brief training period with practice problems. The screen was used only with those six problems that carried in their context the numerals which were to be studied.

Immediately after the screen was placed over one of these problems the subject was asked to give in detail all of the items of the numerals which he could recall. After having proceeded in this manner with the first problem which was to be studied in this way, the subject understood what kinds of details concerning the numerals were desired, and with subsequent problems easily and promptly gave such of the details as he could recall.

The results are reported in tables VII and VIII under five classifications. The classification, complete, signifies that every digit of the numeral was recalled in its true identity and place. The second classification includes correct recall of the digit-length of the numeral and the identity and place of at least its first two digits. The third classification

includes correct recall of the digit-length and the identity of the first digit; and the fourth classification includes correct recall of the digit-length of the numeral. The fifth classification includes those cases in which the subjects were able to recall nothing more of a numeral than its presence in the problem.

2. RANGE OF RECALL OF THE SEVERAL NUMERALS

The range of correct recall from the first readings of each individual numeral is displayed in Table VII. The point which stands out most strikingly after a survey of the table is that almost invariably some item from the first reading of every numeral is correctly recalled. A glance

TABLE VII

RANGE OF CORRECT RECALL OF NUMERALS FROM FIRST READING OF PROBLEMS

	One- and Two-Digit Numerals									Three- to Seven-Digit Numerals									All Numerals
	Problem A		Problem B			Problem F			Problems A, B, F	Problem C		Problem D				Problem E		Problems C, D, E	
Numerals read in problems	43	2	5	45	8	10	9	3	....	246	1754	1000	1276	91	718	2,981,534	617,453	....	....
Total number of readings given each numeral by all subjects	7	7	7	7	7	6	6	6	53	6	6	6	6	6	6	7	7	50	103
Range of correct recall of numerals: Complete	5	6	6	6	7	6	6	6	48	6	1	5	1	0	1	0	0	14	62
First two digits and digit-length	5	..	..	6	..	6	..	..	17	6	4	5	3	0	1	1	1	21	38
First digit and digit-length	6	6	6	7	7	6	6	6	50	6	5	5	4	2	1	6	3	32	82
Digit-length	6	6	6	7	7	6	6	6	50	6	6	5	6	4	3	7	7	44	94
Merely noticed	1	0	0	0	0	0	0	0	1	0	0	1	0	0	2	0	0	3	4

at the "totals" column at the extreme right shows that some item of the numerals in 94 of the 103 total number of readings was recalled. In five cases only, the numeral was not even "merely noticed." Such frequency of recall implies that in the minds of subjects confronted with arithmetical problems to be solved the numerals hold a place of unique significance among the other elements of the problems, and in consequence are noticed almost invariably.

The range of recall which most frequently follows this habitual notice of the numerals also stands out clearly from this table. This item is the correct digit-length of the numerals. It was recalled from a very great majority of the readings, namely, from 94 of the total 103 cases. All of the exceptions occur in Problem 4 where four numerals of

three different digit-lengths occur. In such a situation it was an easy matter for the length of one numeral to be confused with that of another. Evidently the number of digits in a numeral is a point of exceptional interest to the readers. This is not difficult to understand in view of the fact that the number of digits in a numeral is certainly one of the most highly significant indications of its value.

With the shorter numerals, complete recall was achieved in almost every instance. In the left-hand section of Table VII it is shown that such is the case in 48 instances of a total of 53. Apparently no greater demands were made upon the attention of the subjects by the reading of one- and two-digit numerals than practically all of them were both able and willing to meet. It is also apparent that the effort which was habitually used in reading the short numerals so firmly fixed them in memory that they were able to be recalled completely.

The longer numerals on the contrary do not exhibit such large preponderances in the higher ranges of recall as are exhibited by the shorter numerals. In comparatively few instances the longer numerals were completely recalled. The explanation is believed to be due to two facts: first, that longer numerals according to common experience are more difficult to recall than shorter numerals; and second, that in most cases the longer numerals were only partially read during the first reading of the problems.

When reading the longer numerals the subjects evidently gave more emphatic attention to the first one or two digits than to any other digits. In more than half of the cases the first digit was recalled, and in slightly less than half of the cases the first two digits were recalled. In only three instances were three or more digits retained, when recalls of the familiar numeral 1000 and of the numeral 246, which is found in the peculiar place of "first" numeral in Problem C, are excepted. The chief explanation of this greater emphasis on the first digits probably lies in the general habit of attacking printed matter from the left. It is also possible that the adult subjects of this study had learned empirically the greater value of the first digit of a numeral and its greater significance to the solving of the problem, and therefore had formed the habit of paying special attention to it when reading problems.

The first numeral of a problem seems to receive more careful attention during the first reading than any of the other numerals in the problem. More details of the first numeral are correctly recalled than of other numerals. Such a comparison can be made between the longer numerals only, because, as has been pointed out in previous paragraphs of this

study, the shorter numerals almost invariably—and therefore quite without regard to position in the problem—are read closely enough for complete recall. The longer numerals appear in problems C, D, and E. In each of these problems more details of the first numeral are recalled. Problem C will serve as an illustration. Here the first numeral, 246, is completely recalled in all cases, while the second numeral is so recalled only once. In Problem D the numeral 1276 is considered the first numeral rather than 1000 because of the peculiar character of the latter numeral. The higher ranges of recall of first numerals are probably due in part to the advantages of "initial" position. By virtue of this position they would tend to be more vividly impressed upon the memories of the readers. In additon to this advantage it appears probable that the adult subjects of this study have learned empirically to pay greater attention to the first numeral. By so doing they would be able to make special use of it, not only in determining the conditions of the problem, but also as a base in reaching decisions concerning the relations of the numerals to each other.

The peculiar quality of the familiar numeral 1000 again distinguishes it from other numerals of the same length. In every case but one it was recalled completely. This single exception was due to an unusual case of confusion on the part of the subject, which caused him to forget even the sense of the problem.

3. RANGE OF RECALL—BY THE SEVERAL SUBJECTS

Certain subjects recall much more of the numerals than others. The significant differences between them occur in the higher ranges of recall and with the longer numerals. These differences are displayed in detail in Table VIII. The several individuals divide themselves into two general groups according as they reported relatively many or

TABLE VIII

VARYING RANGES OF CORRECT RECALL OF THREE- TO SEVEN-DIGIT NUMERALS BY THE SEVERAL SUBJECTS

	Subjects						
	Hb	R	Bak	Th	L	G	C
Number of numerals read by subjects	8	8	8	4	8	6	8
Range of correct recall of numerals:							
Complete	4	2	2	2	2	1	1
First two digits and digit-length	7	4	4	2	2	1	1
First digit and digit-length	7	7	6	3	3	2	4
Digit-length	8	7	8	4	5	5	7
Merely noticed	0	1	0	0	1	0	1

relatively few of the longer numerals in the higher ranges of recall. Subjects Hb, R, Bak, and Th are included in the first group, and L, G, and C constitute the second group. The contrast between Hb of the first group and G of the second is striking. The former completely recalls half of the longer numerals and the first two digits of seven of the eight longer numerals read, while the latter recalls completely only one longer numeral and the first two digits of only one.

Differences between the two individuals in first-reading attitudes seem to account for the large differences exhibited by them in the ranges of recall. Subject Hb is a pronounced whole first reader. He intends to "grasp" all of a numeral when he first reads it. Subject G, on the other hand, is a striking example of the type of partial first readers. His purpose during the first reading, in so far as the numerals are concerned, is to obtain only a "general idea."

4. FURTHER EVIDENCE AS TO THE PURPOSE OF FIRST READING

Further evidence is found in this study in support of the conclusion presented in the first preliminary study that the main purpose of the first reading is to find the conditions of the problem in order to know how to proceed with solving. In nearly all instances the subjects in this study were able to indicate a correct procedure for the solution of any problem after the first reading. They were able to do this even in the many instances where they could recall nothing more of the numerals than their digit-lengths.

5. ITEMS OF RECALL NOT INCLUDED IN THE CLASSIFICATIONS

The classification scheme used in this study does not include every item concerning the numerals which was reported. In many cases subjects reported correctly the line of the problem in which a numeral appeared. In several cases they recalled its approximate location within the line. No items incorrectly recalled are included in the classifications. Several such items were reported. In general they followed the types of errors which would be found in any study of errors in the reading of numerals in arithmetical problems.

6. SUMMARY OF CONCLUSIONS

The results of the second preliminary study on the range of correct recall of numerals after first reading may be summarized as follows: (1) Some item of almost every numeral is recalled. (2) The digit length of numerals is recalled almost invariably. (3) The shorter numerals and the familiar numeral 1000 are completely recalled almost invariably.

(4) The first one or two digits of longer numerals are recalled in a majority of cases. (5) The first numeral, in problems which include numerals of the greater lengths, is more frequently recalled than any other numeral in the problem. (6) The subjects divided themselves into two groups according as they recalled in the higher ranges large or small proportions of the longer numerals. (7) Further evidence appears in support of the previous conclusion that the main purpose of the first reading of a problem is to learn its conditions.

CHAPTER IV

ANALYSIS OF THE RE-READING OF NUMERALS IN ARITHMETICAL PROBLEMS—THIRD PRELIMINARY STUDY

I. DESCRIPTION OF THE STUDY

The distinction was drawn between the first-reading and the re-reading phases of the reading of arithmetical problems in the first preliminary study. The general purpose of re-reading as stated was "to perceive the numerals accurately for computation." The present study was designed to give further description of the purposes of the subjects and of their activities with the numerals during the re-readings. The general method which was used to obtain the data was that of introspective observation on the part of adult readers.

The readers were four graduate students in the School of Education of the University of Chicago. One of them, Subject S, had read the problems of the first preliminary study. None of the others served as subjects in any other study of the investigation. They were asked to solve the five simple arithmetical problems which were later used as reading materials in the eye-movement studies and which are described in detail in chapter vi. Each subject was given pencil and paper and told that he might use them in solving the problems or not use them, as he chose. Before the beginning of the experiment the subjects were informed concerning the first-reading and re-reading phases of the reading of problems. At the conclusion of the experiment each of the subjects was of the opinion that his reading of arithmetical problems habitually followed these phases, and that the information given concerning them had not caused him to vary from his normal procedure.

The subjects were instructed to attack each problem immediately when it was presented and to proceed with it in accordance with their normal problem-solving attitude. They were to press a conveniently placed telegraph key at the instant of beginning to read and continue the pressure throughout the first reading. Immediately at the conclusion of the first reading the key was released. Thereafter whenever the attention of the subject was directed to the re-reading of any item from the text of the problem, the key was pressed and held, until attention was directed away from the text whereupon the key was immediately released. The effect of this practice was to secure a separate record for each of the one or more acts of re-reading from the text of a problem.

Every pressure and release of the key was recorded on a smoked-paper record sheet which was moving on two kymograph drums. The duration of each pressure on the key was measured in seconds by the use of a chronometer which was so placed that its marker recorded the time intervals on the record sheet side by side with the records from the key. A brief period of training with practice problems in this procedure was necessary in order to enable the subjects to follow the procedure correctly and easily. Immediately after the solving of a problem, and with its text before them for reference, the subjects were asked to report the words or numerals in the text of the problem, upon which their attention was directed at each separate re-reading. This they were able to do with promptness and certainty. The reports of the subjects and the time records from the kymograph are presented in tables IX and X.

The reading of Problem 2 by Subject Ba will serve as an illustration of the experimental procedure. Ba began to read the problem immediately when it was placed before him and at the same moment he pressed the key. The instant he finished the first reading of the problem, which required a time interval of 7.6 seconds, he released the key. Without delay he turned his attention to the numeral 357 in the text of the problem and immediately pressed the key. During an interval of 1.4 seconds he re-read this numeral. He then directed his attention to the sheet of paper on which he intended to copy the numeral and at the same time released the key. When 357 was copied he turned his attention to the numeral 1643, pressing the key at the same instant. During an interval of 2.4 seconds he re-read this numeral. When the re-reading of 1643 was completed he looked to the copy sheet to copy the numeral and at that moment released the key. This done, once more he glanced at the problem, simultaneously pressing the key, and fixed his attention upon the last sentence for .2 of a second. At the conclusion of this interval he released the key and was ready to proceed with solving the problem.

The numerals or words read at each re-reading from the problems are given for every subject in Table IX. The time in seconds required for re-reading the numerals or words is given under the numerals or words in every case. Beginning at the top of the table the first left-hand column reads that two re-readings were given to items from Problem 1 which contains the numerals 47 and 2. The item from the text of the problem which was read by the first re-reading was the numeral 47, and the duration of this re-reading was 2.4 seconds. At the second re-reading the numeral 2 was read and the time required for this second re-reading

TABLE IX

NUMERALS AND WORDS READ AT EACH RE-READING TOGETHER WITH NUMBER OF SECONDS REQUIRED

	Problems															
	1		2			3			4				5			
Numerals read in problems	47 2		357 1643			243,987 21,765			1000; 1276; 91 817				1,918,564; 617,453			
Ordinal number of re-readings	1	2	1	2	3	1	2	3	1	2	3	4	1	2	3	4
Subject Ba																
Numerals and words read	47	2	357	1643	last sentence	243,987	21,765	.....	all of numerals		whole problem	.....	1918	564	617	453
Number of seconds required	2.4	2.4	1.4	2.4	0.2	4.0	4.0	.....	3.0	8.0	7.6	.....	4.6	1.2	4.0	2.6
Subject Gl																
Numerals and words read	47	2	357	1643	.....	243,987	21,765	.....	1000	1276	91	817	1,918,564	617,453	.....	.....
Number of seconds required	0.6	0.2	1.0	1.0	.....	2.0	1.4	.....	1.0	1.6	2.6	1.0	1.8	3.2	.....	.....
Subject S																
Numerals and words read	.....	.....	357 1643	.....	.....	243	987	21,765	1276	91	817	.....	1918	564	617,453	.....
Number of seconds required	.....	.....	1.4	.....	.....	1.4	1.0	1.2	1.2	1.0	0.8	.....	1.6	1.6	0.8	.....
Subject Wm																
Numerals and words read	.....	.....	357 1643	.....	.....	243,987	21,765	.....	"three" and last 3 numerals	1000	last 3 numerals		1,918,564	617,453	.....	.....
Number of seconds required	.....	.....	4.2	.....	.....	4.0	3.4	.....	6.4	2.2	0.8	0.6	5.2	1.6	.....	.....

was again 2.4 seconds. The sum of the durations of the two re-readings from the problem, i.e., the total re-reading time for the problem was, therefore, 2.4 seconds+2.4 seconds or 4.8 seconds. This last detail of information appears in Table X, in the first left-hand column, and in the upper row.

2. OBJECTS AND NATURE OF THE RE-READINGS

The point which stands out most clearly in Table IX is that numerals were the objects of the re-readings almost invariably. Only three instances appear in the entire number of re-readings in which the objects of the re-readings were words. It is also clearly apparent that the numerals of the problems were almost invariably re-read. In five instances only, numerals were not re-read.

These facts serve as additional evidence in support of the conclusion, which was drawn in a previous section, that the numerals are of a nature which clearly distinguishes them from the other contextual elements of arithmetical problems and which causes them to make unusual demands upon the attention of readers.

Further light is thrown upon the procedures of subjects with the numerals by a review of the original reports. These reports show that every numeral which was re-read was copied on the computation paper immediately after it was re-read. In all of these cases paper and pencil were used in solving the problems. In such cases copying the numerals is one of the earlier moves in the total process of solving the problem. Re-reading the numerals appears in most cases to have been a necessary step preliminary to copying them. It is, therefore, proper to speak of such re-reading as re-reading for copying.

The number of readings which was required for the re-reading of a numeral for copying was usually one. Numerals varying in length from one to seven digits were thus re-read at one reading. To none of the numerals of one- to four-digit lengths was more than one re-reading given. To several of the six- and seven-digit numerals, on the other hand, two re-readings each were given. In these cases the first three or four digits of the numeral were re-read and copied as a group, after which the remaining three digits were re-read and copied as a second group. In two instances, the two numerals of Problem 2 were copied from one re-reading.

3. DURATION OF RE-READINGS

The duration of the first reading and the sum of the durations of the re-readings are given for the reading of each of the several problems

by each of the individual subjects in Table X. Examination of the data shows that numerals of greater lengths required greater total re-reading times than numerals of lesser lengths. The total re-reading time in the data for each individual subject increases gradually, in most cases, from the relatively small total time required to re-read the short numerals of Problem 1 to the relatively greater time required to re-read the long numerals of problems 3 and 5. The numerals of Problem 4, which offers four numerals for re-reading, received in most cases a greater total re-reading time than the numerals of any other of the five problems, none of which offers more than two numerals for re-reading.

TABLE X

NUMBER OF SECONDS REQUIRED FOR FIRST READING AND FOR RE-READING OF PROBLEMS

Problems	1	2	3	4	5
Numerals read	47 2	357 1643	243,987 21,765	1000; 1276 91; 817	1,918,564 617,453
Subject Ba—					
First reading time	2.4	7.6	7.4	10.4	9.4
Total re-reading time	4.8	4.0	8.0	18.6	12.4
Subject Gl—					
First reading time	2.0	4.6	6.6	5.8	6.8
Total re-reading time	0.8	2.0	3.4	6.2	5.0
Subject S—					
First reading time	1.8	4.2	7.0	12.8	6.6
Total re-reading time	.....	1.4	3.6	3.0	4.0
Subject Wm—					
First reading time	3.8	5.6	10.6	20.0	10.8
Total re-reading time	.....	4.2	7.4	10.0	6.8

The total re-reading time of a problem is in most cases shorter than the first reading time of the problem. The obvious explanation lies in the comparatively small amount of work to be done during the re-reading. At this time as a rule the numerals only are included in the reading, whereas during the first reading all of the contextual elements of the problem, both words and numerals, are included.

4. SUMMARY

The following conclusions may be drawn from this study: (1) The objects of the re-readings from the problems were almost invariably numerals. (2) The numerals were re-read for copying on the computation sheets. (3) One re-reading for copying was sufficient for most of the numerals. Some of the longer numerals, however, were re-read in two parts. (4) The numerals of greater length required longer times for re-reading, on the part of a majority of the subjects. (5) The total time required for re-reading the numerals was less than the time required for the first reading, with three of the four subjects.

CHAPTER V

READING NUMERALS IN COLUMNS—FOURTH PRELIMINARY STUDY

1. DESCRIPTION OF THE STUDY

The purpose of this study was to give some description of the reading of numerals, when numerals only appeared as the material to be read and when each individual numeral was placed in a separate line. The general plan followed in procuring the data was to have adult subjects copy numerals from the pages on which they appeared onto other sheets, and at the same time articulate the numerals in an easy natural way. This articulation was recorded by the author of this report. By means of a system of notes which will be described later, it was possible to get a fairly full account of what was said and, as experience in making the records accumulated, it was possible to distinguish clearly the various types of reading.

The subjects were four graduate students in the School of Education of the University of Chicago. Subjects R, L, and G had each served in the second preliminary study. Subjects G and H had read the problems of the first preliminary study and photographic records of the eye-movements of both H and G appear in the second part of this report. The materials which they read were forty-eight ordinary whole numerals varying in length from one to seven digits, and including seven numerals for each different digit-length except that there were only six numerals of seven-digit length. Punctuation in the form of commas was used in the customary way with some of the five-, six-, and seven-digit numerals; and with some of these numerals it was not used. Four numerals of both the five- and six-digit lengths, and three numerals of the seven-digit length were punctuated, while three numerals of each of the five-, six-, and seven-digit lengths were not punctuated. The numerals were typewritten on separate lines and so arranged that the tens, hundreds, etc., places of the numeral above were not exactly above the same places of the numeral in the line below. Subjects R and G each read the whole set of numerals twice, while the other two subjects read each set only once.

The subjects copied the numerals from the text sheet on to the copy sheet at normal speed. At the same time they articulated the numerals in an easy low voice which the observer was able to hear at a distance of

approximately two feet. That part of the instructions which called for copying the numerals was inserted in order to provide a genuine working purpose for reading them. At the same time this purpose required an exact reading of every numeral. The kind of reading done, therefore, in compliance with these instructions was of a relatively clearly defined functional type, quite similar in obvious ways to the re-reading of numerals for copying, which was described in the preceding study. The provision for articulation enabled the observer to report the readings in so far as the grouping of the digits of the numerals and the numerical language used in reading them were concerned. At the same time the articulation did not seem to interfere with the reading.

Numerals are said to be read by digit groups when certain successive digits are so closely associated with each other in the reading as to form units of reading, which units are at the same time clearly distinguished from other similar units. The digits which constitute a group are bound together by being pronounced in quick succession as one series. The pronunciations of the several digit groups are separated from each other by time intervals distinctly longer than the time intervals which separate the pronunciations of the individual digits.

For reporting the digit groups and the numerical language used in reading the numerals a simple code system was devised which was based on the symbols 1, 2, and 3 signifying respectively the grouping of the digits of a numeral in groups of one, two, and three digits. A few other signs were necessarily added to indicate various modifications of these groups. A brief period of training with sets of practice numerals was undergone by the observer and by each of the subjects. After this training, they were able to proceed with the experiment in full conformity with the instructions. The data which were obtained during the course of this study are arranged in tables XI–XV.

2. GENERAL DESCRIPTION OF THE THREE MAIN GROUPS USED

The most striking feature of the reading of numerals, which was discovered in this study, was the fact that the subjects habitually divided the numerals into digit groups. Three different sizes of groups were clearly distinguished in the readings, namely, those that were made up of one, two, and three digits respectively. The one-digit groups appeared more frequently in the one-, three-, and seven-digit numerals as is shown in Table XII. The two- and three-digit groups appeared in the readings of numerals of all the greater digit-lengths. Relatively large numbers of three-digit groups appear in readings of the five-, six-,

TABLE XI

MAIN-GROUP PATTERNS USED IN READING ONE- TO SEVEN-DIGIT NUMERALS IN COLUMNS

	DIGIT-LENGTH OF NUMERALS																													
	1		2		3		4		5						6						7									
		F		F		F		F	Punctuated		Non-Punctuated		TF		Punctuated		Non-Punctuated		TF		Punctuated		Non-Punctuated			TF				
										F		F				F		F				F		F						
Main-group patterns.....	1	42	2	42	1-2	25	2-2	34	2-3	24	2-3	15	39		3-3	24	3-3	8	32		1-3-3	16	1-3-3	4	20					
																							2-2-3	4	5					
					3	17	1-3	2			2-2-1	1	1				2-2-2	9	9		2-2-3	1	3-1-3	1	1					
																							1-2-2-2	2	2					
							"one thousand"	6			1-2-2	2	2				1-2-3	1	1				2-1-1-3	1	1					
																							2-1-1-2-1	1	1					
																							2-2-2-1	2	2					
																					1-2-1-3	1			1				
																							3-2-2	1	1					
																							3-4	1	1					
																							1-2-1-2-1	1	1					

F, above, heads columns in which the frequency of appearance of the various main-group patterns is given.

T F, above, heads columns in which the total frequency of appearance of main-group patterns in both punctuated and non-punctuated numerals is given.

and seven-digit numerals. In the cases of the six- and seven-digit numerals the explanation of this condition lies in the fact that numerals of these lengths present just twice as many opportunities for the employment of three-digit groups as numerals of four- or five-digit lengths. In the case of the five-digit numerals, on the other hand, the large number of three-digit groups is attributable to the remarkable uniformity with which the habit of reading five-digit numerals in two groups of two and three digits respectively was followed by all of the subjects.

The three-digit groups exhibited two distinct types which are referred to as the simple and complex types. The three digits of the simple type of three-digit groups are pronounced individually and with equal time intervals between them. The three digits of the complex type on the other hand are pronounced in two distinct subgroups, the first of which includes one digit and the second, two digits. Both types appear in the readings of the five-, six-, and seven-digit numerals, as is seen in the readings of Subject G, which are reported in detail in Table XIV.

Habit on the part of the individual subject appears as the most conspicuous factor in determining which of the two types of three-digit group was chosen, when a three-digit group was used. The last right-hand column of Table XIII discloses the fact that Subject H used the simple type of three-digit group only, while Subject G used it in a preponderant number of cases. On the other hand Subject R used the complex type almost invariably.

3. MAIN-GROUP PATTERNS FOR NUMERALS OF LIKE LENGTH

It became apparent early in the course of this study that the digits of numerals of any one length were being grouped in much the same way by all of the subjects. This observation is strikingly confirmed by the data which are presented in Table XI. The fact that appears most strikingly after an examination of this table is that the digits of numerals of any particular length are divided into a certain number of groups, which groups are made up of certain numbers of digits and stand in a certain order of succession. The case of the seven-digit punctuated numerals will serve for illustration. The digits of these numerals are seen to have been divided into three groups by all of the subjects. In the first group, one digit is found almost invariably, in the second group three digits, and in the third group three digits. To this succession of digit groups of certain sizes only one exception occurs. Such an arrangement of the digits of a numeral is designated as a main-group pattern.

The one- and two-digit numerals were each read as single groups of one and two digits respectively. The first variation from one main-group pattern as representative of the reading of numerals of the same length occurs in the three-digit numerals, which exhibit two patterns.

TABLE XII

NUMBER OF ONE-, TWO-, AND THREE-DIGIT GROUPS USED IN READING NUMERALS OF THE SEVERAL DIGIT-LENGTHS IN COLUMNS

	Digit-Length of Numerals						
	1	2	3	4	5	6	7†
Number of readings given numerals.	42	42	42	36*	42	42	36
Number of:							
One-digit groups	42	.....	25	2	3	1	35
Two-digit groups	.....	42	25	68	45	28	30
Three-digit groups	.....	.....	17	2	39	65	51

* There are 36 readings of four-digit numerals when the 6 readings of the numeral 1000, which was always read as "one thousand," are omitted.

† One four-digit group was used by Subject L in reading one of the seven-digit numerals.

TABLE XIII

NUMBER OF SIMPLE (3) AND OF COMPLEX (1–2) THREE-DIGIT GROUPS USED IN READING FIVE-, SIX-, AND SEVEN-DIGIT NUMERALS IN COLUMNS

	DIGIT-LENGTH OF NUMERALS								
	5		6		7		5–7		5–7
	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated	Punctuated and Non-Punctuated
Subject R—									
Simple three-digit groups	.....	.....	5	.....	7	.....	12	.....	12
Complex three-digit groups	8	6	11	12	5	10	24	28	52
Subject G—									
Simple three-digit groups	6	2	13	2	6	3	25	7	32
Complex three-digit groups	2	3	3	.....	5	.....	10	3	13
Subject H—									
Simple three-digit groups	4	2	8	.....	6	2	18	4	22
Complex three-digit groups	.....	.....	.....	.....	.....	.....	.....	.....	.....
Subject L—									
Simple three-digit groups	1	1	5	3	2	2	8	6	14
Complex three-digit groups	3	1	3	.....	3	.....	9	1	10

The four-digit numerals appear almost invariably in a pattern of two groups of two digits each. A conspicuous exception to this regular main-group pattern of the four-digit numerals is found in the familiar numeral 1000 which was regularly read as "one thousand." This exception is

further evidence of the fact, to which attention has been called in other sections of this report, that this numeral is different in quality from other numerals of the same length.

The five-digit numerals show in a preponderant number of cases a pattern of two groups of two and three digits respectively. The dominant pattern for the six-digit numerals is that of two groups of three digits each. The first main-group patterns made up of three groups to appear among punctuated numerals are found in the seven-digit numerals. When numerals of any of the greater digit-lengths are written without punctuation, other patterns than the main-group pattern for that digit-length make their appearance. In the case of non-punctuated, seven-digit numerals as many as ten different main-group patterns were found. With all of the other digit-lengths, however, conformity to main-group pattern is clearly the rule with all subjects. Evidently such arrangements of the digits of numerals, when they are being read for copying, are procedures which have been very thoroughly conventionalized by long practice, or else such procedures rest closely upon certain fundamental laws of mental action.

4. VARIATIONS IN NUMERICAL LANGUAGE

The fact that all numerals of a certain digit-length were usually read in the same main-group pattern does not mean that the language used in reading them was the same. Variations in language were found in the readings of both two- and three-digit groups. By the use of symbols to represent each of these variations, the detailed numerical language which was used by one subject, namely by Subject G, is given in Table XIV for the readings of all the numerals. The patterns which appear in this table may, therefore, be designated as numerical-language patterns.

Four different numerical-language patterns are found in the readings by Subject G of the punctuated numerals of five-digit lengths. Any five-digit numeral may be pronounced according to each of these patterns. The numeral 76,184, which was one of the numerals read by the subjects, may be taken as an example of the five-digit numerals. When this numeral is pronounced successively according to each of the four numerical-language patterns, as they are represented from top to bottom in the column of punctuated five-digit numerals, the following four different pronunciations result: "seventy-six—one eight four"; "seven six—one, eight four"; "seven six—one eight four"; "seventy-six—one, eighty-four."

TABLE XIV

NUMERICAL LANGUAGE PATTERNS USED BY SUBJECT G IN READING NUMERALS IN COLUMNS

	DIGIT-LENGTH OF NUMERALS									
	1	2	3	4	5		6		7	
					Punctuated	Non-Punctuated	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated
Number of numerals read	14	14	14	14	8	6	8	6	6	6
Numerical language patterns	1	2a 2b	1–2a 1–2b 3b	2a–2a 2b–2b "one thousand"	2a–3b 2b–(1–2b) 2b–3b 2a–(1–2a)	2a–3b 2a–(1–2a) 2a–2a–1	3b–(1–2a) 3b–3b (1–2b–1–2b)	3b–3b 2a–2a–2a 2b–2b–2b	1-(1–2a)(1–2a) 1–3b–3b 1–(1–2a)–3b 2b–2b–3b	2a–1–1–3b 2a–2a–3b 2a–1–1–2a–1 2a–2b–3b 2a–2a–2a–1

Symbols 1, 2, and 3 represent digit groups of one, two, and three digits respectively.

The letter "a" indicates that a digit group is pronounced as one word, i.e., "2a" when representing the numeral 76, indicates that it is pronounced "seventy-six."

The letter "b" indicates that each digit in a digit group is pronounced as a separate word, i.e., "2b" when representing the numeral 76, indicates that it is pronounced "seven six."

The dash, –, separates digit groups.

Parentheses () enclose the two subgroups of complex three-digit groups.

The symbol 1–2a, when representing (any three-digit numeral such as) 694, is pronounced "six, ninety-four."

A second examination of the four numerical-language patterns, the pronunciations of which are presented immediately above, reveals the fact that all four of the patterns are modifications of one fundamental main-group pattern. This fundamental pattern contains two groups, the first of which is a two-digit group, while the second is a three-digit group. It is the variations that appear in the pronunciations of both of these groups that distinguish the four different numerical-language patterns. In the two-digit groups the differences in language are merely those between the words, "seven six" and "seventy-six," or again between "eight four" and "eighty-four." In the three-digit groups the words used to pronounce the simple type differ from those used to pronounce the complex type, as "one eight four" differs from "one, eighty-four."

The habits of individual subjects were discovered in a previous paragraph to be the most conspicuous factors in determining which of the two types of three-digit groups was used. The original records show that such individual habits similarly were the chief factors in determining what language was used in pronouncing the two-digit groups. On the other hand, the fundamental main-group pattern for any length of numeral appeared consistently in the readings of all subjects. It is, therefore, apparent that the selection of the digit-length of groups and the selection of the order of their appearance are more fundamental phases of the reading of the numerals than the selection of the particular type of three-digit group and the choice of the particular words by which the two- and three-digit groups are to be pronounced.

5. INFLUENCE OF PUNCTUATION ON THE GROUPING OF DIGITS OF LONGER NUMERALS

Great differences appear between the readings of punctuated and non-punctuated numerals in the number of both two- and three-digit groups which are employed. Punctuation apparently has the effect of increasing the number of three-digit groups used, and conversely of decreasing the number of two-digit groups. In the extreme right-hand column of Table XV it is seen that each of the last three subjects used a much greater proportion of three-digit groups for the punctuated numerals, and on the other hand a much greater proportion of two-digit groups among the non-punctuated numerals.

The preponderance of three-digit groups in the punctuated numerals and conversely the preponderance of two-digit groups in the non-punctuated numerals are each relatively much greater for the six- and

seven-digit numerals than for five-digit numerals. Such a situation may be partly explained by the fact to which attention was called above, that it is possible to use just twice as many three-digit groups in reading a six- or seven-digit numeral as in reading a five-digit numeral. Fewer main-group patterns appear in the columns for punctuated numerals in Table XI than in the columns for non-punctuated numerals. Examination of the patterns in both columns shows that there are greater numbers of groups in the non-punctuated patterns, and this is due mainly to the more frequent use of the smaller group of two digits.

The readings of one subject, R, exhibited practically no differences in selection of two- and three-digit groups, which may be attributed to

TABLE XV

EFFECT OF PUNCTUATION ON THE NUMBER OF TWO- AND THREE-DIGIT GROUPS USED IN READING FIVE-, SIX-, AND SEVEN-DIGIT NUMERALS IN COLUMNS

	DIGIT-LENGTH OF NUMERALS							
	5		6		7		5–7	
	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated	Punctuated	Non-Punctuated
Subject R—								
Two-digit groups	8	6	.....	.....	.....	3	8	9
Three-digit groups	8	6	16	12	12	10	36	28
Subject G—								
Two-digit groups	8	7	.....	15	2	12	10	34
Three-digit groups	8	5	16	2	11	4	35	11
Subject H—								
Two-digit groups	4	4	.....	9	.....	8	4	21
Three-digit groups	4	2	8	.....	6	2	18	4
Subject L—								
Two-digit groups	4	4	.....	4	1	4	5	12
Three-digit groups	4	2	8	3	5	2	17	7

punctuation. With the larger numerals he used three-digit groups consistently, wherever it was possible to use them, in both punctuated and non-punctuated numerals. The exception in this case is probably attributable to his having attained to a relatively high stage of proficiency in the mechanical processes of reading numerals by means of a large amount of special practice in a kind of reading of numerals which is very similar to that used in this study. This practice he had gained while earning his living in the capacity of railroad rate clerk. A large part of his work was to read numerals and call them off to a colleague, who copied them on other paper.

The easy use of larger digit groups seems to give greater facility and greater speed to the reading of numerals. The value of punctuation

to the subjects in large part lies in the fact that its employment encouraged the use of the larger group of three digits. The subject confronted with the necessity of reading a large but unknown number of unspaced digits is in a difficult situation. Such situations are not frequently encountered in the experiences of the ordinary reader. In consequence he proceeds with caution and with the smaller groups of one and two digits. The great number of small groups and the large number of group patterns which were employed in the readings of non-punctuated numerals of seven-digits length, as shown in Table XI, are evidently results of procedure under difficulty and with uncertainty. The same situation produced the great number and variety of numerical-language patterns in the readings by Subject G of the same numerals. In situations such as these, employment of the symbols of punctuation appears to afford great and immediate relief.

6. PERSISTENCE OF PATTERNS FROM THE FIRST READING THROUGH A SECOND READING

. Opportunity to study the persistence of the main-group and numerical-language patterns, which were found in the first reading of the numerals, through the second reading of the same numerals was given in the cases of two subjects. Subjects R and G each read all of the numerals at two separate readings. The interval of time between the two readings was approximately 30 minutes with each subject. It was found that both R and G read the same numerals in the same patterns at both readings with very few exceptions. Subject R, who was more highly trained in the reading of numerals than any other subject, made fewer changes than G. More changes were made in numerical-language patterns than in main-group patterns. Most of the changes, which were made, were found in the non-punctuated numerals of seven-digits length.

The fact that the same main-group patterns so consistently reappeared at the different readings of these subjects gives further evidence in support of the conclusion, which was advanced in a paragraph above, that the arrangement of digits in main-group patterns has been very thoroughly conventionalized, or else that such procedure rests closely upon certain fundamental laws of mental action.

7. SUMMARY OF CONCLUSIONS

The following conclusions are drawn from the data presented in this study concerning the articulated reading of numerals for copying. (1) The digits of numerals are grouped in the process of reading. The

groups of digits are of three sizes, namely, of one, two, and three digits respectively. (2) The numerals of each of several digit-lengths are read almost invariably in a main-group pattern which is peculiar to that digit-length. (3) Various numerical-language patterns are used in pronouncing numerals of the same length. (4) The employment of punctuation with the longer numerals encourages the use of three-digit groups and, conversely, discourages the use of two-digit groups in the reading. A larger group unit is thus secured. (5) The main-group and numerical-language patterns which are used in the first reading of numerals persist for the most part through a second reading of the same numerals.

PART II. STUDIES OF THE READING OF NUMERALS—BY USE OF PHOTOGRAPHIC APPARATUS

CHAPTER VI

DESCRIPTION OF THE EYE-MOVEMENT STUDIES

1. APPARATUS DESCRIBED

The data which are presented in the remaining sections of this report were obtained through the use of an apparatus designed to record the movements of the eyes in reading by means of photography. The apparatus is described and its use explained in a monograph by Dr. C. T. Gray,[1] and excellent photographs and diagrams of the same are found in a magazine article by Gilliland.[2] A few slight adaptations of the apparatus and procedure, which are described in these references, were necessary in view of the materials and purposes of the present investigation. The materials which were read by the subjects of this investigation were printed on separate cards eight and one-half by four inches in size. These cards were placed on the stand immediately before the lenses of the camera and directly before the eyes of the readers. A flood of light reflected from the overhead mirror gave bright illumination to any materials which were placed upon the stand. A convenient elbow-rest was provided for the right arm in such a manner that computation with a pencil could be undertaken easily and comfortably, and directly upon the problem card, whenever the subject chose to do so. With this arrangement it was possible for the pencil of light which is reflected from the eye, to register continuously upon the film during periods of computation, as well as during periods of reading from the problem.

2. THREE TYPES OF READING-MATERIALS USED

The reading-materials which were selected for this part of the investigation were of three different types, namely, simple arithmetical problems, numerals isolated in lines, and a paragraph of ordinary expository prose.

[1] C. T. Gray, "Types of Reading Ability as Exhibited Through Tests and Laboratory Experiments," *Supplementary Educational Monographs*, Vol. I, No. 5 (1917), pp. 83–91.

[2] A. R. Gilliland, "Photographic Methods for Studying Reading," *Visual Education*, Vol. II, No. 2 (February, 1921), pp. 21–26.

The arithmetical problems were so designed as to provide a simple and genuine problem-setting for the numerals which had been selected for further study. The numerals thus selected included representatives from each of the several lengths of from one to seven digits, the familiar numeral 1000, and a group of three numerals placed closely together in one problem. Further details concerning the problems are given in Table XVI and the problems exactly as they were read by the subjects appear as Selection 1.

TABLE XVI

DESCRIPTION OF THE FIVE PROBLEMS READ IN THE PHOTOGRAPHIC APPARATUS

NUMBER OF PROBLEM	ORDINAL NUMBER OF EACH LINE IN PROBLEM	LENGTH OF LINE IN MILLIMETERS	THE NUMBER IN EACH LINE OF					
			Words	Letters	Numerals	Digits	Words and Numerals	Letters and Digits
1.....	1st	76	9	34	2	3	11	37
2.....	1st	95	11	40	1	3	12	43
	2d	88	8	37	1	4	9	41
3.....	1st	95	11	44	0	0	11	44
	2d	102	7	34	2	11	9	45
	3d	86	9	37	0	0	9	37
4.....	1st	95	10	42	1	4	11	46
	2d	102	8	56	0	0	8	56
	3d	90	8	30	3	9	11	39
5.....	1st	95	8	40	1	7	9	47
	2d	102	10	43	1	6	11	49
	3d	94	12	45	0	0	12	45
Total for all problems..	12	.....	111	482	12	47	123	529
Average number per line	.....	93.33	.....	.....	.....	.....	.....	.....

NOTE.—The number of spaces between words is not counted.

SELECTION 1

FIVE PROBLEMS READ BEFORE PHOTOGRAPHIC APPARATUS

1. At 47 cents a dozen what will 2 dozen eggs cost?

2. A timber man owns one plot of 357 acres, and another of 1643 acres. How much ground does he own altogether?

3. A wholesale grain firm had at the beginning of the day 243,987 bushels of wheat. During the day's trading 21,765 bushels were sold. How much did they then have?

4. A commission house had on hand 1000 cases of canned corn. From three different canning factories they bought respectively 1276, 91, and 817 cases. How many did they then have?

5. If one telephone company uses 1,918,564 cross bars during the year, and another company in the same period uses 617,453 cross bars, how many more does the one use than the other?

The numerals isolated in lines included a list of thirty-four numerals. Twenty-eight of the list of thirty-four consisted of ordinary numerals which were selected by taking four numerals from each of the seven-numeral lengths of one to seven digits. In addition, the list included the six special form numerals, namely, 1000, 333, 25,000, 0, 99, and 637,637. They were presented to the readers on two different cards. The line space between any two numerals was 16 mm., and the lines were placed two spaces apart. These two details of arrangement were followed in order that the reading of any numeral might be entirely separate from the reading of any other. The numerals are reproduced as Selection 2 and in the same form in which they were presented to the readers.

SELECTION 2

Isolated Numerals Read before Photographic Apparatus

(Card One)

836	3	5489	756,352	46	4,325,986
85,974	239	1	16,789	1024	354,908
12	2,374,957	1000	333	25,000	

(Card Two)

76,184	9317	17	2	5,236,795	256
743,819	1928	365	8	93,548	3,984,673
107,308	52	0	99	637,637	

The selection of ordinary expository prose was taken from Judd's *Psychology of High-School Subjects*. The subjects read directly from the book. Data were tabulated from the reading of ten lines by each subject. Because of defects in the records of subjects W and G only five and seven lines respectively were tabulated from their readings. The regularity in number and duration of pauses found in the data for the few lines, which were tabulated for these subjects, however, give evidence that the data for these few lines represent the normal reading of these subjects in these materials. The record which represented the reading of Hb was totally unsatisfactory for use. The ten lines of the text were each 93 mm. in length and included 101 words and 452 letters. They are reproduced as Selection 3.

SELECTION 3

ORDINARY PROSE READ BEFORE PHOTOGRAPHIC APPARATUS

"Anyone who has struggled with the German language has an appreciation of the satisfaction which the novice feels in watching the way an expert in this language manages a separable verb. The moment the verb is used in a sentence, there arises a feeling of craving for the remainder of the verb. The skillful German places between the verb and the prefix a long series of phrases and words, but ultimately arrives with perfect precision at the end of the sentence, and gives the satisfaction which comes from a proper closing of the feeling which was started when the...."[1]

3. INSTRUCTIONS TO SUBJECTS AND DESCRIPTION OF SUBJECTS

The instructions given the readers were varied for each of the three kinds of materials which were read. When the problems were being read the subjects were asked to attack them with the normal problem-solving attitude. Each individual was provided with a pencil which he was told he might, or might not, use in computation, as he chose. The instructions which the readers received for the isolated numerals were to read the numerals successively in the lines, to read all of them accurately, and to proceed at the normal rate of speed. Each numeral was to be articulated in an easy, natural manner and in a voice which was barely audible to the observer, who stood at a distance of approximately three feet. Provision for this slight articulation was included as a means of encouraging the complete reading of all numerals. For the expository prose selection the instructions were to read silently for a clear under-

[1] C. H. Judd, *Psychology of High-School Subjects*. Boston: Ginn & Co., 1915. Pp. 190.

standing of the paragraph and at normal speed. The volume from which the selection was taken was familiar in a general way to all of the readers. They were given its title in advance and the subject-matter of the passage to be read was described as relating to the psychology of language.

The six subjects, records of whose readings appear in this part of the investigation, were all male graduate students in the School of Education of the University of Chicago. Three of them had served in various preliminary studies. Subject G had read the problems of the first and second preliminary studies and was classified as a pronounced partial first reader of numerals in problems. Subjects H and Hb had served in the first and second studies respectively and were both found to be pronounced whole first readers. The original plan of the investigation specified that the subjects whose types of reading had been studied in the preliminary sections should act as subjects for the eye-movement experiments. Of the photographic records which were made of the subjects of previous studies, however, only those of G, H, and Hb were entirely satisfactory. None of the subjects reported past experiences which seemed likely to have had important influence upon his reading of the materials of this study.

4. PROCEDURE ON THE PART OF THE OBSERVER

The instructions were given to the subjects before they took their seats at the camera, and samples of each of the three kinds of materials, which were to be read, were examined by them. When the readers were properly seated before the camera they were given a brief training with practice problems and with practice sets of isolated numerals until procedure according to the instructions was mastered. After the solving of each problem, several of the subjects were asked to make brief introspective observations concerning whole and partial first reading of numerals, the re-reading of numerals and the steps used in computation. Their reports were recorded and later served as a basis for interpretation of the corresponding eye-movement records.

5. GUIDE FOR READING THE PLATES

The photographic films, upon which the lines of dots representing pauses of the eyes were recorded, were used as slides in a projection lantern. The records of the photographs were in this manner projected upon a screen, which, at the same time, held the texts of the various reading materials. The photographic picture of the subject's reading was thus superimposed upon the exact text of the materials which he had read. It was, therefore, possible to locate directly upon the lines of the

text itself the letter or digit about which the attention of the subject was centered at any pause of the eye. By counting the number of dots in the lines of dots, which represented the pauses, the exact durations of the pauses were ascertained.

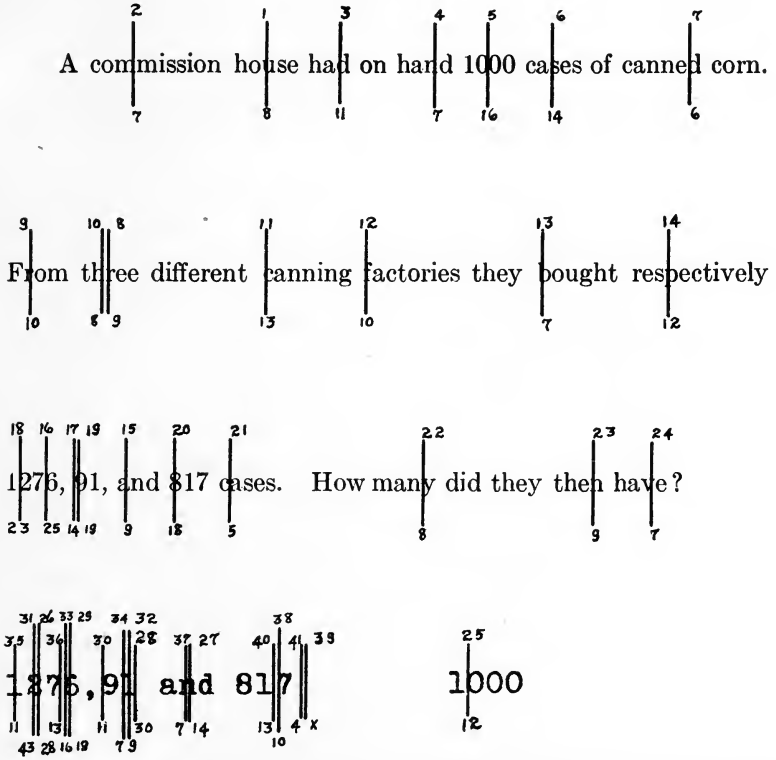
The plates, which describe readings of the problems, are numbered I–XV as presented in this chapter. The remaining plates, which describe readings of the isolated numerals, are numbered XVI–XXV and are found in chapter ix. The lines of reading materials which are found in the plates are reproductions of the lines which were read by the subjects. The short straight vertical lines which cross the lines of print represent pauses of the eye. The particular letter, digit or space which is crossed by a vertical line represents the approximate center of the field of perception which was included in that pause. The arabic numbers, 1, 2, 3, etc., above each of the vertical lines indicate the serial order of each pause among the pauses which were used in reading the problem. When the serial number of a pause moves to the left of the serial number of the previous pause a backward or regressive movement of the eye is indicated. The number at the lower end of a pause line gives the duration of the pause in units of 1/50 of a second.

The vertical lines which mark the pauses used in the first reading of the problem are located on the lines of the reproduced text. All pauses used in re-reading or in copying numerals or in computation are recorded below the last line of the problem. For convenience in interpretation the numerals which were read during such pauses are typewritten below the last line of the problem and directly below the several digit spaces occupied by these numerals in the lines below. A straight horizontal line below the vertical lines that indicate the computation pauses, describes the location of numerals which have been copied, or of answers which have been recorded.

The reading of the plates may be illustrated by the reading of Plate XII, which is as follows: Pause 1, which falls in the first line of the problem, begins the first reading of the problem. It is located on the letter "a" of the word "wholesale" and the duration of the pause is 10/50 of a second. Pause 2, which is a backward or regressive movement from Pause 1, is located on the letter "o" of "wholesale" and the duration of this pause is also 10/50 of a second. There are seven pauses in line 1. Pause 9, which is in line 2, falls on digit "9" of the numeral "243,987" and its duration is 20/50 of a second. It is a regressive movement from Pause 8. Pause 22 completes the first reading of the problem.

The pauses which follow represent the process of computation and are recorded below the text of the problem. Pause 23, which is the first pause used in the process of computation, is apparently a locating pause. It is followed by Pause 24, by which digit "7" of the numeral

PLATE I



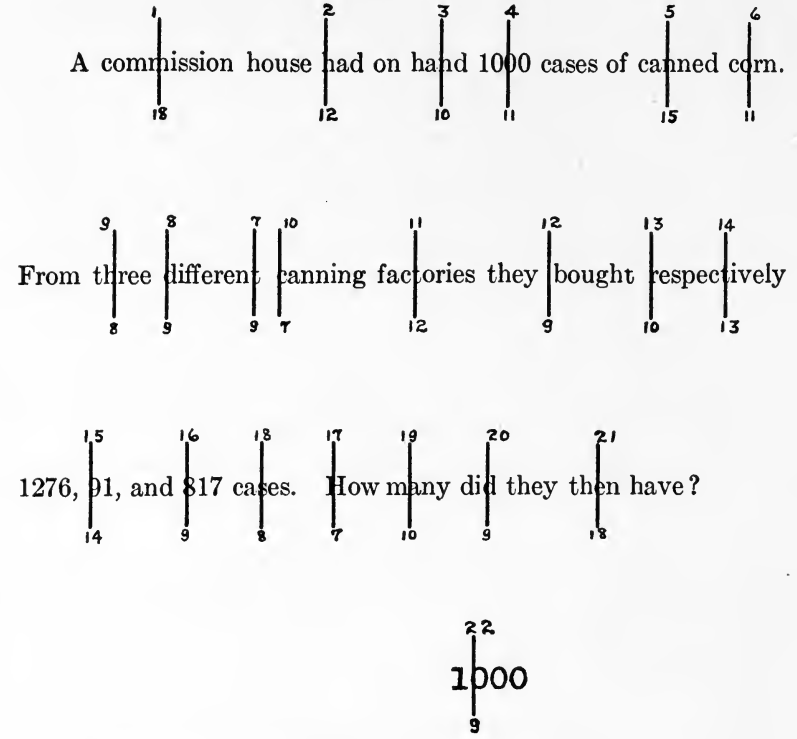
First reading of Problem 4 by Subject B and his procedure in solving the problem. *x* indicates that it was impossible to determine with precision the duration of the pause.

"243,987" is read in 41/50 of a second. Attention then passes immediately with Pause 25 to digit "5" of "21,765," which digit is to be subtracted from the afore-mentioned digit "7." Pauses 26, 27, etc., continue the process of computation, which ends with Pause 32. The subject then shut his eyes and the record was finished.

Plate I records the reading of Problem 4 by Subject B. In the first reading of the problem, which is represented by the pauses in the lines

of the text itself, an initial regression is noted in line 1. Pause 1 evidently was not located closely enough to the left end of the line for a satisfactory beginning. Similar initial regressive movements appear in lines 2 and 3. During pauses 16 to 19, inclusive, the two numerals 1276 and 91 were given whole first readings. With Pause 24 the first reading of the problem was finished.

PLATE II

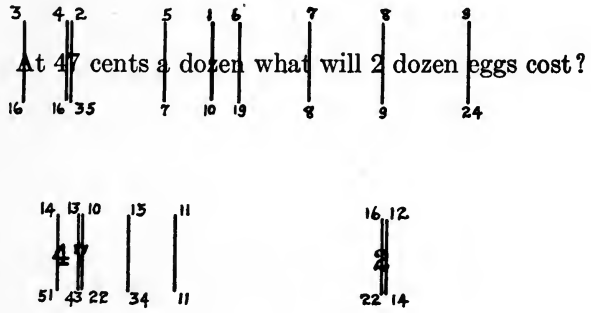


First reading of Problem 4 by Subject G and re-reading the numeral 1000

The remaining pauses, which represent subsequent procedure with the problem, are placed below the lines of the text. The numeral 1000 was re-read with Pause 25. The records do not give sufficiently detailed information for the identification of the purposes of the individual pauses subsequent to Pause 25. The computation was, however, conducted directly from the problem card and apparently the subject added the numerals 1276 and 91 first, and then added 817 to that result. The answer was recorded during, or immediately after, Pause 41.

Plate II contains the record of Subject G for Problem 4. He proceeded rapidly with the first line, but read the second line with a number of pauses which is relatively large as compared with the number of pauses on his other lines. The numeral 1000 was read in detail with Pause 4 in a time interval of 11/50 of a second. With pauses 15 and 16 he gave partial first readings to the three numerals, 1276, 91, and 817. Such partial first readings of these numerals were in this instance sufficient preparation for computation with them. The first reading of the problem was completed with Pause 21.

PLATE III



*The answer, 94, was recorded during Pause 16 at the point indicated.

First reading of Problem 1 by Subject W and multiplication direct from the problem card with one numeral used as the "base of operations."

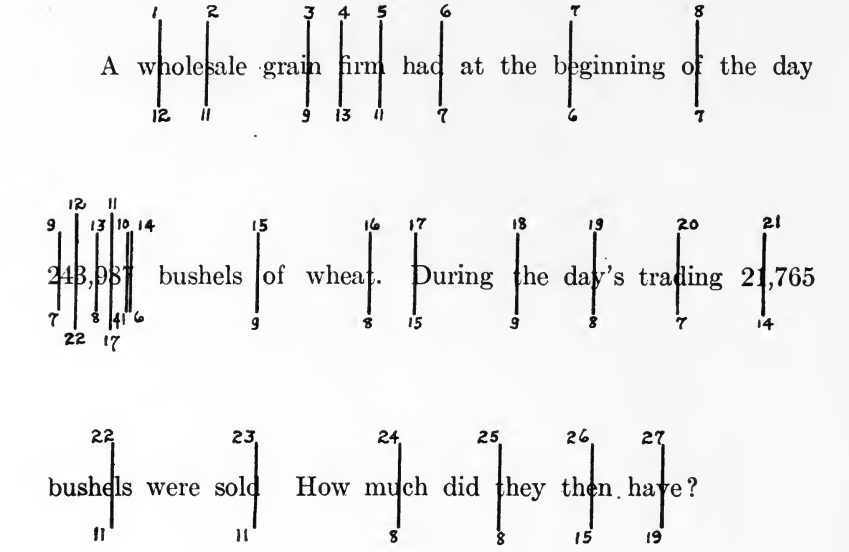
Immediately after the first reading, the subject quickly re-read 1000. After Pause 22, the record was unsatisfactory.

Plate III shows the first reading of Problem 1, the re-reading of both of the numerals, and the process which was followed in solving the problem. The first pause fell much too far to the right of the beginning of the line and two regressive movements were necessary as shown by pauses 2 and 3. The numeral 47 was evidently carefully read in the two pauses of 16/50 and 35/50 of a second which it received. The first reading of the problem was completed with Pause 9.

The re-reading began immediately with Pause 10 on the numeral 47. It was a long pause of greater than average duration notwithstanding the fact that 47 had been carefully read during the first reading.

Pause 11 probably served as a guiding pause in the long move from 47 to 2. This last numeral was re-read during Pause 12. After this the subject returned to the numeral 47 and made it the base of the operation

PLATE IV



The numerals were copied at the place indicated by the horizontal line.

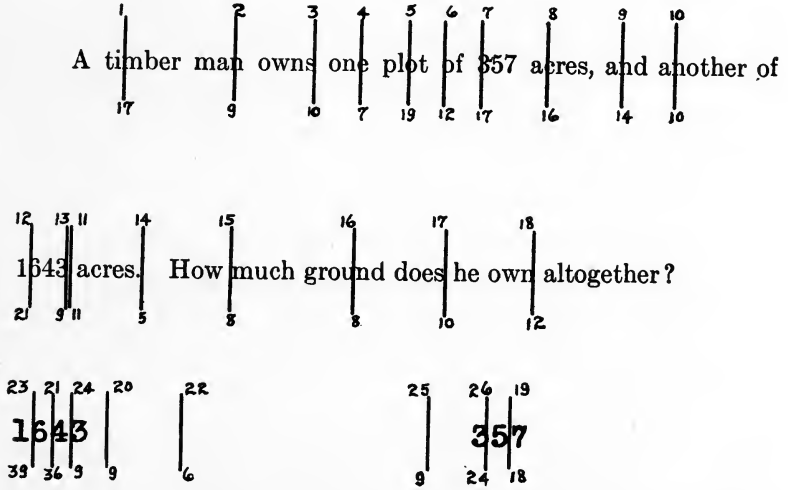
First reading of Problem 3 by Subject Hb and re-reading the numerals for copying. *x* indicates that it was impossible to determine with precision the duration of the pauses.

of multiplication, which took place during pauses 13 and 14. When the computation was completed, the eye returned to the vicinity of the numeral 2, where the answer was recorded during Pause 16.

Plate IV illustrates the first reading of Problem 3 and the re-reading of both of its numerals for copying by Subject Hb. The numeral 243,987 was given a detailed whole first reading, while the numeral 21,765 in the same line was passed by with a rapid partial first reading. Of the large number of pauses used in reading line 2, six were placed on the digits of one numeral. The first reading was concluded with a relatively rapid reading of the last line.

Immediately after the first reading, pauses 28 and 29 were used apparently in locating the first numeral and Pause 30 in locating the

PLATE V



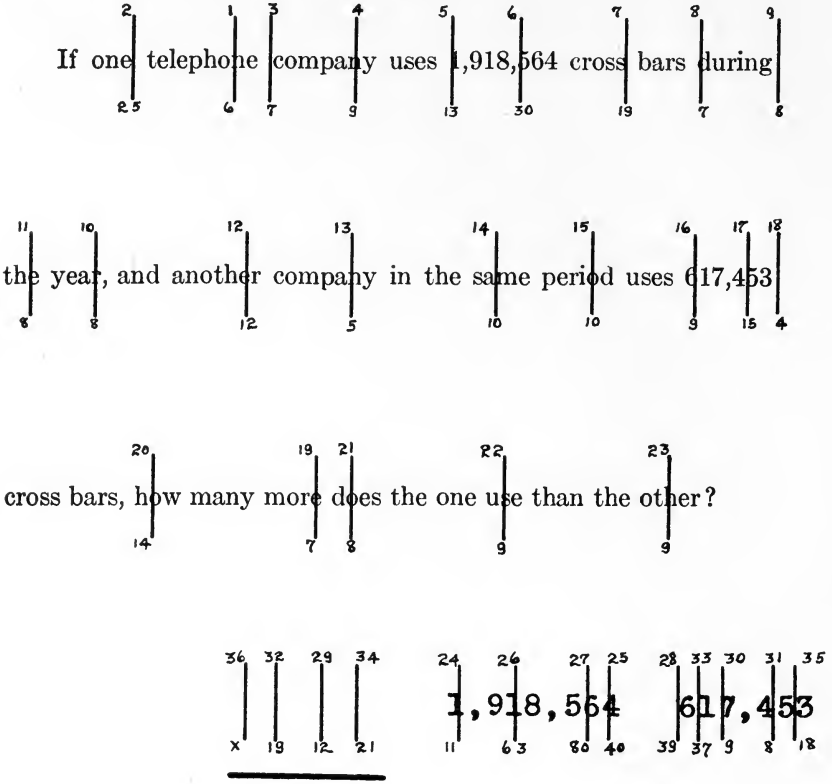
First reading of Problem 2 by Subject Hb, re-reading the first numeral and subsequent re-reading both numerals for copying.

place where it was to be copied. During pauses 31 and 32, the numeral was re-read. During pauses 33 and 34 reference was again made to the place of copying, while during pauses 35 and 36 the numeral was located again. With pauses 37 and 38 apparently the first group of digits was re-read, and the second group of digits was re-read with Pause 40. The second numeral appears to have been located with pauses 42 and 43, and it was then re-read. During Pause 45 or Pause 49 (or during both pauses) 21,765 was copied. After Pause 49, the record could not be followed accurately.

Plate V shows that Subject Hb gave a very detailed and cautious first reading to Problem 2. Ten pauses, none of which represented a

regressive movement, were required to read the first line. In the second line, the numeral 1643 was given a whole first reading with pauses 11, 12, and 13.

PLATE VI



The numerals were copied at the place indicated by the horizontal line.

First reading of Problem 5 by Subject M and re-reading and copying the two numerals. *x* indicates that it was impossible to determine with precision the duration of the pause.

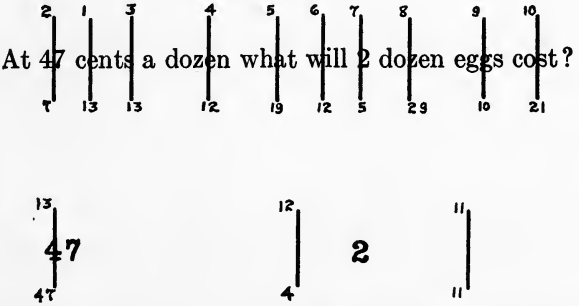
When the first reading was finished Hb proceeded immediately to re-read 357, apparently moved by some such purpose as the re-location of the numeral or the verification of one of its digits. He then re-read and copied 1643 with pauses 20 to 24, inclusive, whereupon he passed to 357, which he re-read with Pause 26. After this pause, the subject's attention was directed to the margin at the left of the text of the problem.

During the first reading of the first two lines of Problem 5, as shown in Plate VI, Subject M used a large number of pauses of relatively short durations. The last line of the problem was read with much greater rapidity. Each of the long numerals was given a partial first reading. M approached the second numeral with a short pause and left it with another short pause. This method of reading long numerals was followed by him in the case of isolated numerals in several instances to which attention is called in the comment on plates XX–XXI.

Immediately at the end of the first reading he re-read the numeral 1,918,564 with pauses 24 to 27, inclusive, and copied it at the same time at the point indicated without moving his eyes from the numeral. The second numeral was re-located with Pause 28, and the first numeral which had now been copied was located with Pause 29 in order to determine where to copy the second numeral. This numeral was then re-read and copied at the place indicated in the plate.

Plate VII shows Subject Hb reading the first problem very cautiously. With only two exceptions every word and numeral in the problem was

PLATE VII



First reading of Problem 1 by Subject Hb and re-reading the first numeral for copying.

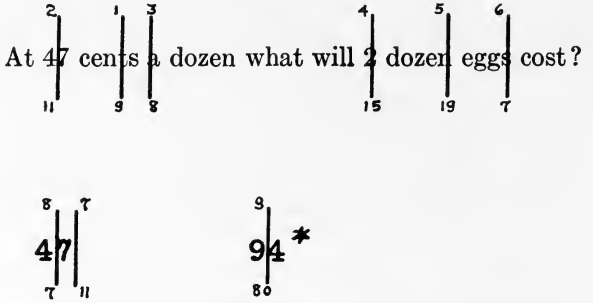
read individually. Such a large number of pauses is in sharp contrast with the relatively small number of pauses which were used by B and G in reading the same problem as shown in plates VIII and X.

After the first reading, which was finished with Pause 10, the numeral 2 was not re-read. The first numeral, however, was re-read with Pause 13 and immediately copied on the problem card in the margin to the left of the text.

In Plate VIII a rapid first reading of the text of the problem by Subject B is observed. Immediately at the conclusion of the first reading,

the subject proceeded to the first numeral which was used as the "base of operations" for the multiplication process in pauses 7 and 8. The numeral 2 meanwhile was retained in memory. The answer was recorded during Pause 9 immediately below the word "what" in the text of the problem.

PLATE VIII



* The answer, 94, was recorded at the point indicated during Pause 9.

First reading of Problem 1 by Subject B and the process of computation with the first numeral as the "base of operations."

In Plate IX Subject G is shown reading Problem 2 with a relatively small number of pauses. Only four pauses were used in the last line. The first numeral was read partially while the second numeral, 1643, was read in detail. G is the only subject who, when the isolated numerals were being read, was able to read four digits in detail with one pause. The instances in which he did this are described in the comment concerning plates XVI and XVII.

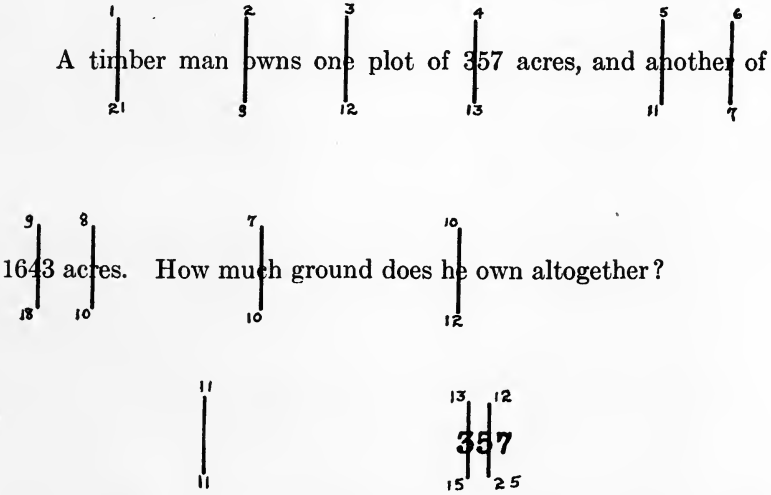
When the first reading was completed, the subject turned his attention to the numeral 357 and used it as the "base of operations" during the process of addition. This operation was carried on "mentally" and directly from the problem card. One or two digits were taken at a time, the computation starting from the right with Pause 12. The answer was recorded during Pause 13 or immediately thereafter. At the end of Pause 13 the subject closed his eyes.

The plate itself supplies ample internal evidence of the fact that the numeral 1643 was wholly read with the first pause which was 18/50 of a second in duration. Such is undoubtedly the case since the subject was able to produce the correct answer without ever looking at the numeral again.

The reading and solving of Problem 1 by Subject G is described in Plate X. The conditions of the problem and the identity of the numerals were evidently grasped during the first five or six pauses. The numerals were not re-read and the answer was recorded during either Pause 8 or Pause 9, or during both.

During pauses 6 and 8 the subject may have been occupied with "mental" computation. This suggestion is offered as a possible explanation of the fact that Pause 6 was not located on any reading material, but was nevertheless the longest of all pauses used in connection with the

PLATE IX



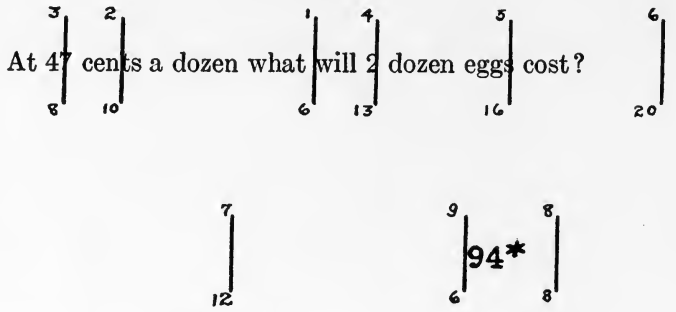
First reading of Problem 2 by Subject G and the process of adding the two numerals.

problem. It does not seem probable that Pause 7 was needed by this subject as a re-reading pause in this problem. If computation was proceeding during pauses 6 and 7, evidently the eyes were roving around without direction. Such undirected roving occurred very rarely, if at all. In most cases, the eyes of the subjects were fixed on the numerals which were involved in the computation.

The first reading, the re-reading, and the solution of Problem 1 by Subject M are shown in Plate XI. Apparently the subject became confused on the first few words of the line as is indicated by the backward and forward movements of the pauses. Such confusion in the reading of problems was found in but very few instances.

After the first reading, some of the last words of the problem and the numeral 2 were re-read. Such re-reading of words in a problem was a very rare occurrence on the part of the subjects of this investigation. The answer was recorded immediately after Pause 18 in the margin of the card to the left of the text of the problem.

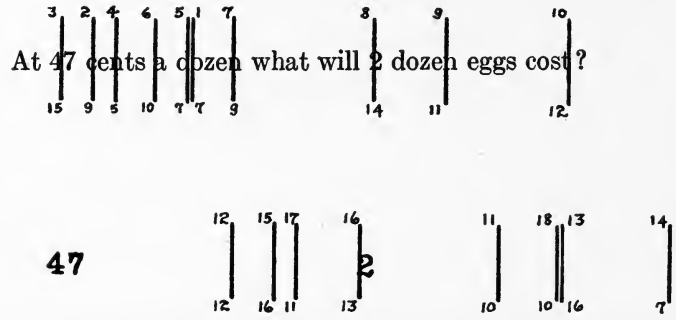
PLATE X



* The answer, 94, was recorded during Pause 8 or 9.

First reading of Problem 1 by Subject G and the process of computation

PLATE XI

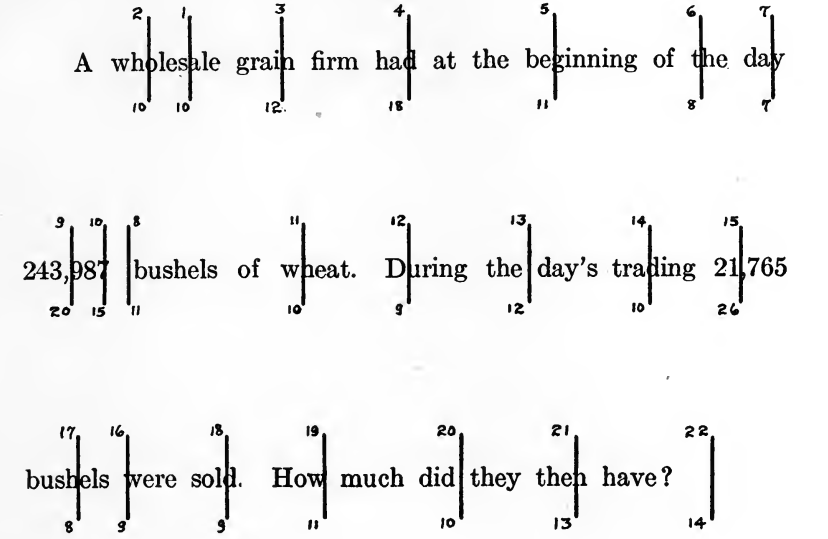


First reading of Problem 1 by Subject M, re-reading words and the process of computation.

During the first reading of Problem 3, as shown in Plate XII, Subject G gave both of the numerals partial first readings. Immediately at the end of the first reading, which was finished with Pause 22, he began the process of subtracting the second numeral from the first. With Pause 23 he located the first numeral and with Pause 24 perceived its first right-hand digit. He then quickly glanced at the first right-hand

digit of the second numeral with Pause 25. The movements back and forth between the two numerals continue steadily, one-digit place being computed at each movement, until the answer was recorded immediately after Pause 32.

PLATE XII

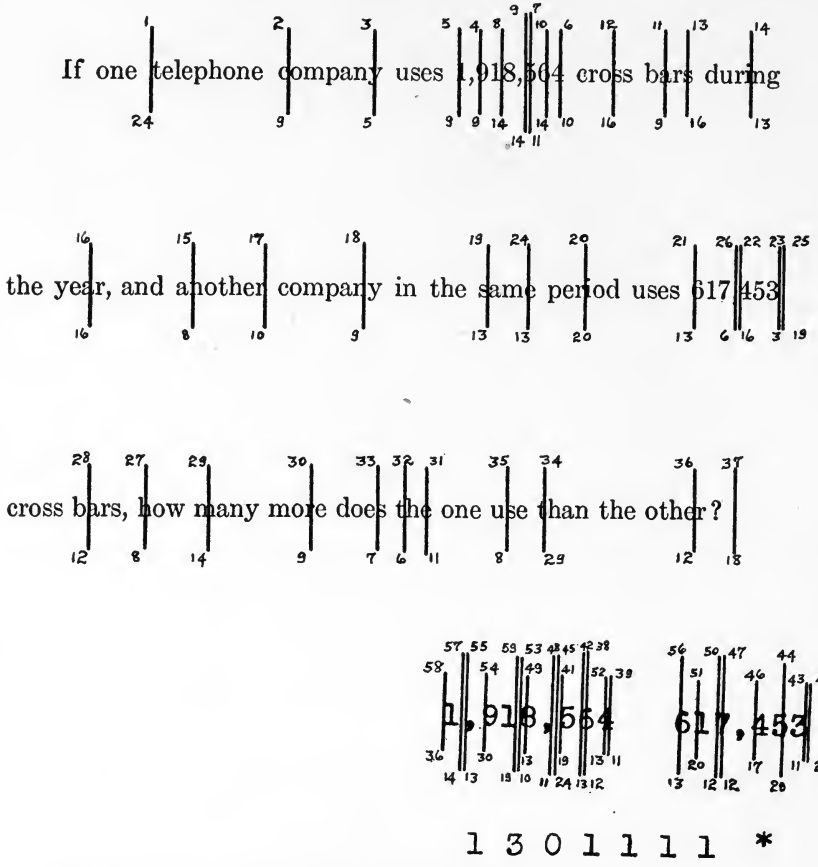


First reading of Problem 3 by Subject G and the process of computation

Attention should be called to the fact that since all of the digits of the answer were the digit 2, it was easier for the subject to hold the answer in memory as long as he did before recording. The larger numeral is seen to have given one more pause, not counting Pause 23, and the average duration of its pauses was greater than that of the smaller numeral. The computation began and ended with the digits of the longer numeral.

In Plate XIII are found illustrations of pronounced whole first reading of numerals by Subject H. Even the text of the problem seems to have been read and re-read with very short spans of attention and with meticulous care.

PLATE XIII



*The answer, 1301111, was recorded at the point indicated.

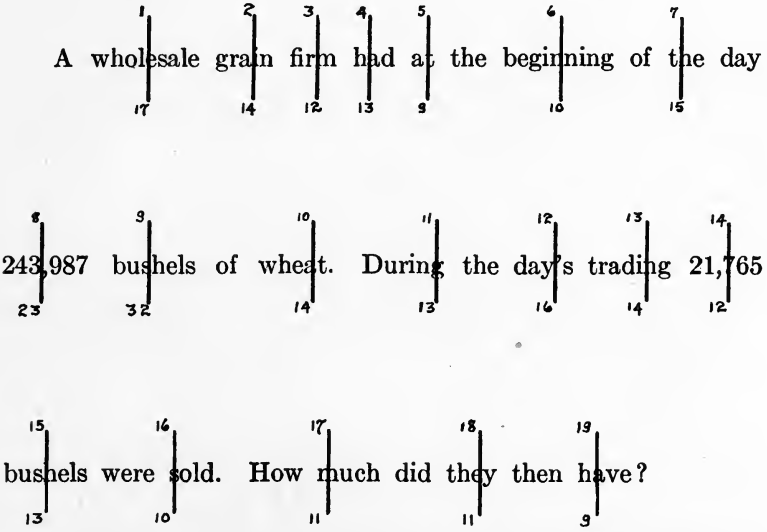
First reading of Problem 5 by Subject H and the process of computation

After the first reading, which is concluded with Pause 37, the computation began immediately and proceeded in a manner similar to that described in the comment concerning Plate XII. The numeral 1,918,564 was used as the "base of operations"; the computation both began and ended with its digits.

The figures of the answer, 1,301,111, were recorded one digit at a time, as they were produced by the computation, and immediately below the words, "use than the other," in the text of the problem. Several of the pauses were used in directing the hand as it recorded the digits of the answer. This was true of pauses on each of the two numerals. An effort is made to give the numbers of such pauses in Table XXVI.

In Plate XIV two excellent cases of pronounced partial first readings are found. Although the numerals are five and six digits in length, respectively, nevertheless, each one was read with a single pause. In

PLATE XIV



First reading of Problem 3 by Subject W with partial reading of numerals

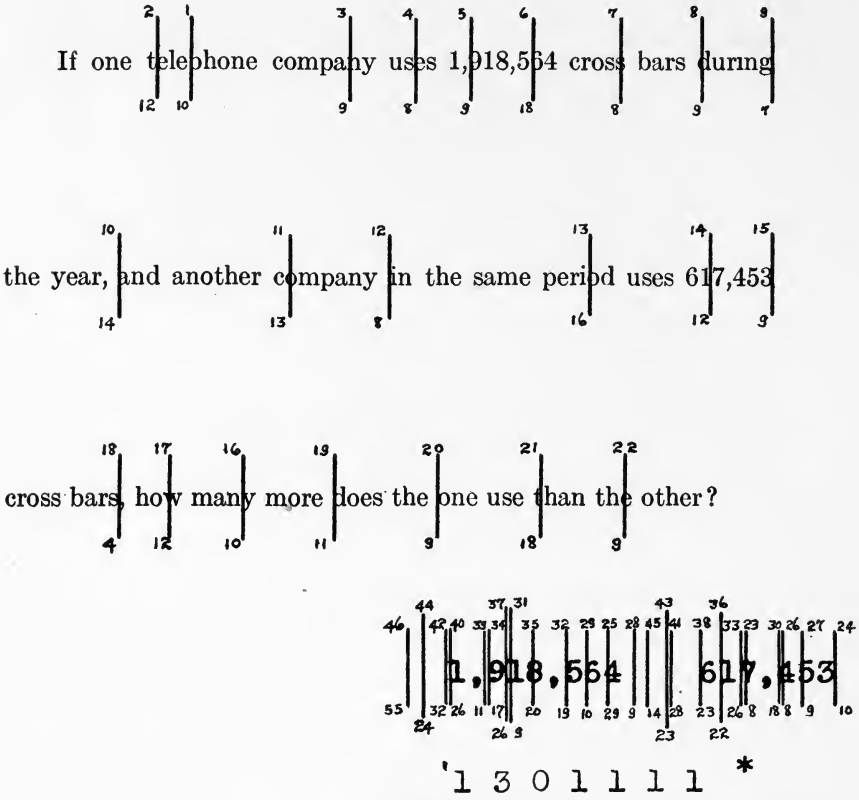
this plate a clear illustration of rapid reading of the last line of a problem is also found. Only five pauses were required for reading line 3 and their durations were less than this subject's average pause-duration on words as given in Table XVIII.

Plate XV exhibits the process of solving Problem 5 as it was carried on by Subject G. His procedure was similar to that of Subject H which is described in the comment accompanying Plate XIII.

The use to which Subject G put each individual pause in the computation is described in Table XXVI. An important difference should be noted between the procedures of subjects G and H in solving Problem 5.

Subject G, as is shown by the location of pauses 23 and 24 of Plate XV, began the computation by taking the first right-hand digit of 617,453 and proceeded to relate it to the corresponding digit of 1,918,564. He continued the process by moving from right to left. Subject H, on

PLATE XV



*The answer, 1301111, was recorded at the point indicated.

First reading of Problem 5 by Subject G and the process of computation

the other hand, began the computation by taking the first right-hand digit of 1,918,564, and proceeded to find the corresponding digit of 617,453. He continued the process, as did Subject G, by moving from right to left. Both subjects, however, appear to have emphasized the larger numeral as the "base of operations." The details are given in Table XXVI.

This concludes the general description of the photographic records. In the following divisions of the report various phases of the reading of numerals will be discussed in greater detail. In the next chapter a description is given of the first reading of the problems material. Chapter viii provides a discussion of the re-reading of problems and the processes of computation. The reading of isolated numerals is described in chapter ix. In the last chapter the performance of the subjects of this investigation is compared with that of the subjects of an important investigation by another author, and finally the report concludes with a discussion of the differences in the demands which are made upon the attention of readers by the three different types of reading-materials.

CHAPTER VII

FIRST READING OF NUMERALS IN PROBLEMS

1. INTRODUCTION

When examining plates I–XV, which reproduce the lines of the problems as they were read and which locate within the lines the pauses as they occurred in the readings, the unusually large number of pauses per line stands out very conspicuously. The average number of pauses per line for all subjects is 8.08; and there are individual lines in which as many as 10, 11, 12, 13, and even 14 pauses are found. The large number of pauses appears all the more remarkable when it is remembered that all of the readers were advanced graduate students, who are entirely familiar with simple arithmetical problems, and who would be expected to qualify as better than average readers.

Attention should be called at this point to the fact, which is given more detailed treatment in a later section, that the subjects of this study were not slow readers. It appears that there is good ground for assuming that the reading of arithmetical problems is more difficult than the reading of ordinary prose. The question suggests itself, therefore: Did the two elements of which the problems are composed, namely, the numerals and the accompanying words, make equal demands upon the attention of the individuals who read them in this study? The data, by means of which comparisons may be drawn between the numerals and the words, with respect to average duration of pauses, average number of letters or digits included per pause, and the percentage which the regressions are of the total number of pauses, are presented in tables XVII–XIX.

2. COMPARISON BETWEEN THE READING OF NUMERALS AND WORDS IN PROBLEMS

It is evident from a glance at Table XVII that there is a very great difference in the average ranges of acts of perception according as digits in numerals or letters in words are read. In the readings of all of the subjects the average number of digits included by a pause on numerals was less than the average number of letters included by a pause on words. The disparity between these averages is slightly greater in the cases of the three whole first readers B, H, and Hb, all of whom show shorter

ranges of perception of digits than the three other subjects, who are partial first readers. Even the partial first readers, however, in every case, perceived on the average less than half as many digits as letters per pause.

The explanation of this shorter range of perception for numerals than for words, when both occur in the same arithmetical problem is probably the same as that given by Dearborn in accounting for the short "number span of attention," which he had noted.[1] The digits in numerals do not appear constantly in the same combinations as do the letters in words. In consequence, the numerals in their continually new combinations of digits make larger demands upon the attention of readers. Every individual digit is significant in itself and must be noted; and all of the digits must be viewed in combination before the numeral

TABLE XVII

AVERAGE NUMBER OF DIGITS INCLUDED IN A PAUSE ON NUMERALS CONTRASTED WITH AVERAGE NUMBER OF LETTERS INCLUDED IN A PAUSE ON WORDS DURING FIRST READING

	SUBJECTS						AVERAGE FOR ALL SUBJECTS
	G	M	W	B	H	Hb	
Average number of digits included in a pause on numerals	3.40	2.35	2.93	1.81	1.88	1.88	2.38
Average number of letters included in a pause on words	7.30	5.95	6.79	7.90	5.18	5.74	6.47

NOTE.—Each subject read the five problems which included 12 numerals totaling 47 digits, and 111 words totaling 482 letters.

is completely read. Words, however, as several investigations of the span of perception have shown, are perceived as wholes. The letters appear and reappear in the same regular combinations, which become familiar in the earlier years of schooling. Readers have become accustomed to them as words and are able to proceed easily with whole words as units of perception.

As noted in the foregoing paragraph, the pauses on numerals are more concerned with analysis and combination of the component digits than the pauses on words are with similar processes with the letters. Such a difference would be expected to make itself evident in a greater average duration for the pauses on numerals than for the pauses on words. The data which are displayed in Table XVIII justify such an

[1] W. F. Dearborn, "The Psychology of Reading, An Experimental Study of the Reading Pauses and Movements of the Eye," *Columbia University Contributions to Philosophy and Psychology*, Vol. XIV, No. 1 (1906), pp. 70–71. New York: The Science Press.

expectation. With each of the several subjects, it is seen that the average duration of the pauses on numerals was decidedly greater than the average duration of the pauses on words. The average for all subjects of the average pause-durations, when numerals were read, is approximately 40 per cent greater than the same average duration when words were being read.

TABLE XVIII

AVERAGE DURATION OF PAUSES IN FIFTIETHS OF A SECOND ON NUMERALS CONTRASTED WITH AVERAGE DURATION OF PAUSES ON WORDS DURING FIRST READING

	Subjects						Average for All Subjects
	G	M	W	B	H	Hb	
Total number of pauses used by subject in reading { Numerals..	14	20	16	26	27	25	.....
Total number of pauses used by subject in reading { Words....	66	81	71	61	93	84	.....
1. Average duration of pauses on numerals	13.92	15.20	15.31	18.46	13.30	13.48	14.98
Average variation	3.92	4.54	5.35	5.92	3.97	5.0	4.77
2. Average duration of pauses on words	10.72	9.87	13.02	9.18	11.74	10.99	10.92
Average variation	2.27	1.41	3.38	1.16	4.13	2.94	2.55

A comparison between the numerals and the words in respect to the percentage which the number of regressive pauses is of the total number of pauses for a subject, yields further evidence of the greater reading-demands made by numerals. In the cases of subjects M, B, H, and Hb, as found in Table XIX, decidedly larger percentages of regres-

TABLE XIX

PERCENTAGE OF REGRESSIVE PAUSES ON NUMERALS CONTRASTED WITH PERCENTAGE OF REGRESSIVE PAUSES ON WORDS DURING FIRST READING

	Subjects					
	G	M	W	B	H	Hb
Total number of regressive pauses located by subject on { Numerals....	2	6	1	7	10	5
Total number of regressive pauses located by subject on { Words.....	11	10	4	8	22	1
Percentage which the number of regressive pauses on: 1. Numerals is of the total number of pauses on numerals.....	14.28	30.0	6.25	26.9	40.0	20.0
2. Words is of the total number of pauses on words	16.6	12.3	5.63	13.1	23.65	1.19

sive pauses appear in the case of the numerals than upon the accompanying words. The explanation of such differences probably lies in the difficulty of reading in the same lines, materials which call for such different ranges of attention and durations of pauses as did the numerals and the words in these problems. When proceeding at the rate of reading and with the range of perception which is adapted to words,

the subject apparently passes over some of the numerals with a reading which does not satisfy him, and he immediately returns to read or to re-read all or a part of the numeral.

3. PARTIAL AND WHOLE FIRST READING OF NUMERALS

Whole first reading was defined in the first preliminary study of the investigation to include such readings of numerals during the first reading of a problem as noted the character of the numeral and the identity and place in the numeral of each individual digit. Any reading of a numeral which did not include these items was called a partial first reading. In Table XX the kind of reading given each of the twelve numerals in the problems by each of the several subjects is described in detail. The data which are included in this table are based upon introspective observations concerning their readings by several of the subjects, and upon inferences which were drawn directly from the plates. With the longer numerals a very small number of pauses of short duration in some instances gave indisputable evidence of partial reading. In several of the records answers to problems which included shorter numerals were computed and recorded when the numerals had been read only on the first reading; obviously such readings were whole first readings. A few readings could not be placed with certainty in either category and are, therefore, marked D, which means doubtful. Plates II, IV, VI, XII, and XIV present instances in which numerals received partial first readings, and plates I, IV, IX, and XIII show other instances in which numerals received whole readings.

There are marked differences between partial and whole first readings of the longer numerals in respect to the number of pauses per numeral and the total time required for reading the numeral. These differences are quickly apparent when Table XX and the plates which bear illustrations of the two methods are studied in detail. Illustrations of both methods of reading are found in Plate IV which represents the reading of Problem 3 by Subject Hb. In this problem one of the numerals, 243,987, was given a whole first reading which included six pauses and measured a total reading-time of 101/50 seconds, while the other numeral, 21,765, was given a partial reading which included only one pause with the duration of 14/50 of a second.

Further emphasis is given to the difference between partial and whole reading of numerals by a comparison of the first readings of the numerals in the problems with the readings of the numerals isolated in lines. As prescribed by the conditions, the readings of the isolated numerals

TABLE XX

DURATION IN FIFTIETHS OF A SECOND, AND SERIAL ORDER OF THE SEVERAL PAUSES USED IN WHOLE AND PARTIAL FIRST READINGS OF NUMERALS

Subjects	Problem Number...	1		2		3		5		4				All Problems		
														Number of Subjects		
	Numerals which were read in problems..	2	47	357	1643	21,765	243,987	617,453	1,918,564	1000	1276	91	817	Partial Readings	Whole Readings	Doubtful Cases
G	Durations of pauses	13	8	13	18	26	20, 15	12, 9	9, 18	11	.....	14	9	7	4	1
	in serial order.....	(W)	(W)	(P)	(W)	(P)	(P)	(P)	(P)	(W)	(P)	(D)	(P)			
M	Durations of pauses	14	15	12, 21, 19	22, 23	10, 20, 8	9, 17	9, 15, 4	13, 30	14	14	.....	16	6	5	1
	in serial order.....	(W)	(W)	(W)	(W)	(D)	(P)	(P)	(P)	(W)	(P)	(P)	(P)			
W	Durations of pauses	9	16, 35	9	19	12	23	9, 9	15, 13, 24	13	14	7	18	9	3	.....
	in serial order.....	(W)	(W)	(P)	(P)	(P)	(P)	(P)	(P)	(W)	(P)	(P)	(P)			
B	Durations of pauses	15	11	18	19	25, 18	8, 45, 9, 18, 14	12, 34, 14, 14	8, 9, 20, 18, 36	16	25, 23	14, 19	18	1	8	3
	in serial order.....	(W)	(W)	(D)	(P)	(D)	(W)	(W)	(W)	(W)	(W)	(W)	(D)			
H	Durations of pauses	14	19	7, 33	11, 14, 14	11, 10	12, 17, 7	13, 16, 3, 19, 16	9, 9, 10, 11, 14, 14, 14	15	.....	17	20	4	7	1
	in serial order.....	(W)	(W)	(W)	(W)	(P)	(P)	(W)	(W)	(W)	(P)	(D)	(P)			
Hb	Durations of pauses	5	7	17	11, 21, 9	14	7, 41, 17, 22, 8,	11, 23	10, 9, 13, 11	12	13, 38	12	20, 15	2	7	3
	in serial order.....	(W)	(D)	(D)	(W)	(P)	14 (W)	(P)	(D)	(W)	(W)	(W)	(W)			
All Subjects	Number of:															
	Partial readings....	.....	.....	2	2	4	4	4	3	.....	4	2	4			
	Whole readings....	6	5	2	4	.....	2	2	2	6	2	2	1			
	Doubtful cases....	.....	1	2	.....	2	.....	.....	1	.....	.....	2	1			

NOTE.—The capital letters, W and P, in the columns above are abbreviations for whole and partial first readings, respectively. The letter D is used to designate doubtful cases which the records do not clearly place in either of the other classifications.

were of the quality of whole readings. The data are arranged in Table XXI for such a comparison between the two sets of numerals in each of the several digit-lengths with respect to the average number of pauses per numeral, the average pause-duration, and the average time required for reading individual numerals. In respect to each of the three points named for comparison the isolated numerals are found to have larger averages than the problem numerals in each of the several digit-lengths. If only those problem numerals which were given partial first readings were included in the comparison, the differences between the two sets of numerals would be appreciably greater than they are.

On the other hand a certain degree of qualification is attached necessarily to the significance of the large differences found because of the effect probably produced on the reading of the isolated numerals by one of the conditions under which they were read. The slight articulation used in reading the isolated numerals presumably acted to diminish the speed with which they were read. It should be noted also that all of the subjects gave evidence of greater interest in reading and solving the problems than in reading the numerals isolated in lines. The probable effect of such great interest in the problems was to stimulate the readers to a more rapid rate of reading the numerals in problems than the numerals of corresponding digit-length which were isolated in lines.

Proper allowances should be made for the differences which were produced in the readings of the two sets of numerals by such variations in conditions as are named above. When this allowance is made, it is found that the whole first readings which were given the longer numerals in the problems by the whole first readers include numbers of pauses and total reading-times per numeral which are similar to those found in the readings of the isolated numerals. The whole first reading of numerals in problems is evidently similar in kind to the reading of numerals isolated in lines.

As was found in the first preliminary study, marked differences appear between the shorter and longer numerals in respect to the number of times in which they were partially and wholly read during first readings. The one- and two-digit numerals were read in detail in most instances in the present study as they were in previous studies. Only one pause, for the most part, was required for the reading of the shorter numerals and the durations of such pauses ran as low as 5/50 and 8/50 of a second. The longer numerals on the other hand received a slightly greater number of partial readings than whole readings. The partial

TABLE XXI

FIRST READING OF NUMERALS IN PROBLEMS CONTRASTED WITH THE READING OF ISOLATED NUMERALS OF CORRESPONDING LENGTHS

(Time unit = 1/50 of a second)

Digit-length of numerals	1		2		3		4		5		6		7	
Number of numerals in problems and of numerals isolated in lines which were read by each of the six subjects	In Problems: 1	Isolated: 4	In Problems: 1	Isolated: 4	In Problems: 1	Isolated: 4	In Problems: 1	Isolated: 4	In Problems: 1	Isolated: 4	In Problems: 2	Isolated: 4	In Problems: 1	Isolated: 4
Average number of pauses per numeral	1.0	1.15	1.17	1.20	1.50	1.90	1.83	2.40	1.66	2.90	3.08	3.70	3.83	4.15
Average pause-duration	11.66	19.30	15.71	22.86	16.44	27.04	16.46	26.28	15.40	28.43	11.46	26.88	14.64	25.82
Average time required for reading individual numerals	11.66	21.45	18.5	28.7	24.66	48.05	30.16	59.65	25.66	77.75	35.17	94.05	56.16	104.54

NOTE.—The numerals of Problem 4 were not included with the numerals of the other problems when the averages presented in the table were computed.

readings for the five-digit numerals included one pause for the most part; and in the six- and seven-digit numerals two pauses were included in most instances. In respect to proportion of partial readings received, the compact group of three numerals in Problem 4, which are placed one immediately after the other, may be classified with the longer numerals.

The familiar numeral 1000 has received a kind of reading which distinguishes it from other numerals of the same length, wherever it has appeared in the preliminary studies. In the present study 1000 was given whole first readings without exception. In no instance was more than one pause required for the reading, and the lengths of the pauses closely approximate average pause-durations. For whole first readings of other numerals of four-digit length, which are found in the problems, two or three pauses were required. Unlike the other four-digit numerals, 1000 is evidently read as a whole, as words are read.

It is probable that the form of 1000, which is that of the digit "1," followed by the very obvious group of three "0" digits is easier of perception than any ordinary group of four digits. It is probably easier of perception than any numeral which is made up of a group of four digits all of which are the same digit. The frequency of appearance of 1000 in the experience of the average reader is probably greater than that of any other single combination of four digits. By virtue, therefore, of the obviousness of its structure and by virtue of the frequency of its use the numeral 1000 has become familiar to the average reader in the same sense that words are familiar, and in consequence is read as words are read.

The number of digits which it is possible to read partially in one pause of partial reading is relatively large. The five digits of the five-digit numerals were read with one pause in three of the four instances when they were partially read. One of the six-digit numerals was read one time with one pause, as is shown in Table XX. The six and seven digits of the six- and seven-digit numerals were, however, read with two pauses in most instances when they were partially read. The usual number of digits read at one pause when such longer numerals were read partially is, therefore, either three or four.

In the preliminary study which was concerned with the range of recall from the first reading of numerals in problems it was shown that in a great preponderance of instances the subjects were able to recall at least the number of digits in a numeral. In view of this fact it seems reasonable to assume that one or more of the subjects who read the

five-digit numerals partially and with a single pause, were able to read all five of the five digits at one pause, at least to the extent of noting that a numeral was there and that its length was five digits. Apparently, therefore, it is possible for subjects to read to this extent as many as five digits at one act of perception.

4. THE SEVERAL SUBJECTS AS PARTIAL AND WHOLE FIRST READERS

The largest factor in determining whether the longer numerals shall be given partial or whole first readings is found in the attitude of individual subjects toward these numerals. The same numerals were given partial readings by some of the subjects and whole readings by others. An examination of the data in Table XX makes it evident that some of the subjects gave partial readings with noticeable consistency to the longer numerals, while other subjects with approximately equal consistency gave whole readings to these numerals. Subjects G, M, and W clearly exhibit the former tendency and may, therefore, be classified as partial first readers, while B, H, and Hb illustrate the latter tendency and may be called whole first readers.

5. RELATIVE VALUE OF PARTIAL AND OF WHOLE FIRST READING

Any determination of the relative value of whole and partial reading as methods of reading numerals during the first reading of a problem will be concerned to a large extent with the question as to which of the methods is more economical of the reader's time. This question resolves itself in large part into a comparison between the partial and whole first readers in respect to the total time required to read all of the numerals of the problems. The three partial readers G, M, and W, as is readily seen by inspection of Table XXII, used decidedly shorter total times

TABLE XXII

READING OF NUMERALS IN PROBLEMS BY PARTIAL FIRST READERS CONTRASTED WITH READING OF NUMERALS IN PROBLEMS BY WHOLE FIRST READERS

(Time unit = 1/50 of a second)

	Subjects					
	Partial First Readers			Whole First Readers		
	G	M	W	B	H	Hb
Total time required to read all numerals	195	304	245	480	359	337
Total number of pauses required to read all numerals	14	20	16	26	27	25
Average pause-duration	13.92	15.20	15.31	18.46	13.30	13.48

NOTE.—Each subject read all five problems which included twelve numerals.

for the first reading of all of the numerals than the whole readers B, H, and Hb. Partial first reading, therefore, in so far as time required for reading the numerals is concerned, was undoubtedly the more economical of the two methods.

The words of the problems were also read more rapidly by the partial first readers, although the case is not as clear for the words as it is for the numerals. The details are given in Table XXIII. The fastest reader of the words is the whole first reader, Subject B, who is also the slowest reader of the numerals. His case appears to be very exceptional and no adequate explanation is found in the records, nor was the subject himself able to account for his relatively high speed with words. After B, the two partial readers, G and M, have the fastest records. The slowest record was made by the whole first reader H. By virtue of relatively higher speeds with both numerals and words the partial first readers completed the first reading of the problems in shorter total reading-times than the whole readers.

TABLE XXIII

READING OF WORDS IN PROBLEMS BY PARTIAL FIRST READERS CONTRASTED WITH READING OF WORDS IN PROBLEMS BY WHOLE FIRST READERS

(Time unit = 1/50 of a second)

	SUBJECTS					
	Partial First Readers			Whole First Readers		
	G	M	W	B	H	Hb
Total time required to read all words of problems...	708	800	925	560	1092	923
Total number of pauses required to read all words of problems.....	66	81	71	61	93	84
Average pause-duration.....	10.72	9.87	13.02	9.18	11.74	10.99

NOTE.—Each subject read all five problems which included 111 words.

Another basis may be found upon which significant inferences can be drawn as to the relative value of the two methods of first reading of numerals. It is possible to draw a comparison between the several individuals, who use the one or the other of the two methods, in respect to their rates of reading with reading materials other than arithmetical problems. Stated in other terms the point in question is: Which of the two methods was used by the readers who exhibit higher rates of reading in other materials. The data concerning the rates of speed with which the subjects read the ordinary prose selection may be used in this comparison. With these materials the average reading-time per

line for subjects M and G were 44.88/50 and 52.52/50 seconds, respectively, and for subjects W, H, and B they were 75.21/50, 75/50, and 80.78/50 of a second, respectively. It is clear, therefore, that the faster readers of the ordinary prose selection used the partial method of reading numerals on the first reading, and two of the three slower readers of the ordinary prose used the whole method of reading numerals.

The more rapid reading of the partial first readers is due to the fact that for the most part they used fewer pauses in reading the same materials than the whole readers. Such is the explanation of the greater speeds whether the materials which were used were the numerals or words of a problem, or the lines of the ordinary prose selection. A smaller number of pauses also explains the exceptionally short total reading-time of Subject B on the words of the problems.

6. DEVELOPMENT OF THE METHOD OF PARTIAL FIRST READING

The large differences between the methods of partial and whole first reading in the several important respects to which attention has been directed in this immediate study and in previous studies of the investigation can be satisfactorily explained only in the light of large differences in attitude on the part of the individuals who use respectively the one or the other of the two methods. The evidence is strong that partial and whole first readers entertain very different attitudes toward the numerals when the problem is being read for the first time. The fact that such differences between the two groups do exist does not necessarily imply that the members of either group are conscious, either of their own peculiar attitude, or of the existence of a different attitude. Most of the subjects of the investigation were not conscious of their attitudes. The implication is, rather, that through long experience with problems these subjects have developed in an empirical way two widely differing sets of habits of reading numerals during the first reading of a problem.

When first learning to read numerals in problems, these subjects, as most individuals probably do, proceeded to read the numerals very slowly, as they came to them, and one digit at a time. The whole reading attitude toward numerals would, therefore, be the attitude more natural for beginners in reading arithmetical problems. In the course of extensive experience, apparently, some of the individuals were more impressed than others, with the differences between words and numerals in respect to the rates of speed which were found practicable in reading them. These individuals were thus stimulated to learn a new

method of procedure with the numerals, which would make for a quicker disposition of them in reading problems. Such a new procedure could not consist simply of a radical increase of the span of perception to include larger units of recognition, as had been done with word materials when whole words and phrases came to be taken as units of perception. Perception of the larger numerals by numeral wholes appears to be quite impracticable because of the nature of numerals as continually varying combinations of digits.

The plan that was learned consists of a rapid passage over the numeral during which time details are skipped and only the most outstanding facts concerning it are gathered. Such is the procedure which is designated as partial first reading and in preliminary sections of the investigation the identity of the first digit of a numeral and recognition of the number of its digits were found to be facts of the outstanding nature referred to. Various ranges of partial first reading came to find empirical acceptance in the course of the development of the habit of partial reading by the natural trial-and-error method of learning. Such changes in procedure were probably able to be accomplished with little or no embarrassment to individuals in the practical solving of problems because of the almost invariable habit of re-reading the numerals after the first reading of a problem.

7. SUMMARY OF CONCLUSIONS

The chapter may be summarized as follows:

1. The numerals of problems make greater demands upon the attention of readers than do the accompanying words, as is shown by the following facts: (*a*) The average number of digits included by a pause on numerals is decidedly smaller than the average number of letters included by a pause on words. (*b*) The average duration of pauses on numerals is greater than the average duration of pauses on words in the cases of all subjects. (*c*) The percentage of regressive pauses on numerals is greater than the percentage of such pauses on words. The explanation of the greater demands of the numerals probably lies in the fact that the combinations of digits in numerals are continually different, whereas combinations of letters in words remain stable.

2. More pauses per numeral and greater total reading-times per numeral are required for whole first reading of numerals than for partial first reading.

3. Shorter numerals are given whole first readings almost invariably. Longer numerals, on the other hand, are given whole or partial readings according as the subjects who read them are whole or partial readers.

4. The numeral 1000 is regularly read as if it were a word rather than a numeral.

5. The subjects divide themselves into groups of partial first readers and of whole first readers according as they read the longer numerals by the partial or whole method.

6. The partial method is more economical than the whole method in point of total time required to read the numerals during the first reading of a problem.

7. The subjects who read the ordinary prose selection more rapidly use the partial method of reading the longer numerals for the most part.

8. The more rapid rates of reading in both the words and numerals of problems and in the ordinary prose selection are exhibited by subjects who use the smallest number of pauses in such readings.

9. Partial first reading of numerals is probably learned empirically as a method of more rapidly disposing of the numerals in reading. The essential characteristics of the method are skipping the details of a numeral and recognizing only the most outstanding facts concerning it.

CHAPTER VIII

RE-READING AND COMPUTATION

1. TWO TYPES OF RE-READING OF NUMERALS

Re-reading from a problem takes place immediately upon completion of the first reading. Two distinct types of re-reading of numerals appear. The two types are distinguished by differences in function. In some cases such differences are disclosed directly by reports which were made by the subjects themselves upon the basis of introspective observations as to what numerals were re-read and why they were so re-read. In other cases definite evidence as to which type of re-reading was used is found by careful examination of the readings as they are described in the plates.

The first type, which may be called simple re-reading, has as its function, apparently, the gathering of further information concerning the numerals before a decision has been reached as to what plan will be followed in solving the problem. Only one of the numerals of a problem is re-read after this fashion. A single case of exception was found in the reading of Problem 5 by Subject Hb when two numerals were re-read in this way. The implication is that such re-reading is undertaken only when the subject is definitely interested in some specific detail of a certain numeral. According to the reports of subjects these specific details include various items of verification of numerals such as the identity of certain digits, the number of digits in the numeral, and the location of the numeral within the line.

The type of re-reading which was given a numeral is indicated in Table XXIV in all cases but two. The two exceptions are with the readings of problems 2 and 3 by Subject M. In these instances several of the words of the problems were re-read along with the numerals. Because of this fact interpretation of the records was complicated to such an extent as to make it impossible to distinguish with certainty which type of re-reading was given the numerals.

Instances of simple re-reading are described in detail in plates I, II, III, V, and XI. The numeral 1000 was given such a re-reading by four of the subjects; illustrations are found in plates I and II. Subject Hb gave a re-reading of the simple type to numerals in each of the last four problems, and an illustration of his procedure in the case of

TABLE XXIV

TYPE OF RE-READING GIVEN TO NUMERALS, OR TO NUMERALS AND WORDS, BEFORE BEGINNING OF COMPUTATION

Subjects	Problem				
	1	2	3	4	5
Hb	Re-read and copied 47	Re-read 357 Re-read and copied 1643 Re-read and copied 357	Re-read 243,987 Re-read and copied 243,987 Re-read and copied 21,767	Re-read 1000 Re-read and copied 1276 Re-read and copied 91 Re-read and copied 817	Re-read 1,918,564 Re-read 617,453 Re-read and copied 1,918,564 Re-read and copied 617,453
M	Re-read some of the words and the numeral 2	Re-read some of the words and numerals and copied the numerals	Re-read some of the words and numerals and copied the numerals	Re-read 1000 Re-read and copied 1276, 91,817 Re-read and copied 1000	Re-read and copied 1,918,564 Re-read and copied 617,453
W	Re-read 47 and 2	None	None	None	None
H	None	None	None	None	None
B	None	None	None	Re-read 1000	None
G	None	None	None	Re-read 1000	None

NOTE.—Two types of re-reading are distinguished in the table. (1) Simple re-reading, such as was given the numeral 243,987 in Problem 3 by Hb, is designated by "Re-read 243,987." (2) Re-reading for copying, which was done in the case of the numeral 1643 in Problem 2 by Hb, is designated by "Re-read and copied 1643." "None" in the columns signifies that no re-reading was done.

Problem 2 appears in Plate V. In Plate XI is found an instance of simple re-reading of the numeral 2 along with certain words of the problem by Subject M.

The second type of re-reading of numerals is that for the purpose of copying the numerals on to the problem card. In the case of each re-reading so classified the records show that the subject after he had re-read the numeral, proceeded immediately to copy it on the problem card. Such re-reading was the normal procedure for subjects Hb and M as is shown in Table XXIV. More detailed descriptions of re-readings of this type are found in plates V, VI, and VII. The record of Subject M, which is presented in Plate VI, shows a distinct variation from the procedure of re-reading as described above. Subject M re-read the numeral 1,918,564 by the four pauses, 24 to 27, inclusive, and at the same time copied the numeral in the vicinity of pauses 32 to 34, without moving his eyes from it during the process of re-reading and copying.

Whether a numeral was or was not re-read depended upon the habits of individual subjects rather than upon either the length of the numeral or upon the quality of first reading which it had received. Numerals of all digit-lengths, including both those which had received whole first readings and those which had received partial first readings, were re-read. Re-reading was practiced systematically both by subjects Hb and M, the former of whom was classified as a whole first reader and the latter as a partial first reader. Of the four subjects who almost invariably proceeded without re-reading, two were classified as whole first readers and two as partial first readers.

Attention should be called at this point to the marked differences between the small proportion of re-readers of numerals, as reported immediately above, and the fact that all subjects re-read most of the numerals of the problems which were used in the first preliminary study. Subjects G and H who with only one exception did not re-read the numerals in the present study did, however, persistently re-read the numerals of the problems of the first preliminary study. In subsequent paragraphs, re-reading is found to be closely connected with the procedure of copying the numerals on paper for computation with pencil. With only occasional exceptions this procedure was followed regularly by all of the subjects in the first preliminary study, including subjects G and H. It is probable, therefore, that their persistent re-reading in the first study was done for the most part in order to insure accuracy in copying the numerals on the computation paper.

2. METHODS AND PROCEDURES USED IN THE PROCESS OF COMPUTATION

Two distinct methods of computation were exhibited by the subjects in respect to their procedure with the numerals immediately after the first reading of a problem. In one case two of the subjects re-read and copied the numerals on the problem card, and wrote out on the card with pencil the figures used in the process of computation. In the other case, four of the subjects computed "mentally" and directly from the numerals as they were printed on the problem card. The former may be called computation from copied figures and the latter, computation direct from the problem card. Table XXV shows which procedure was followed with each problem by each of the several subjects.

TABLE XXV

TWO METHODS OF PROCEEDING WITH NUMERALS FOR THE PURPOSES OF COMPUTATION AFTER THE FIRST READING

Subject	Problem				
	1	2	3	4	5
Hb.....	X	X	X	X	X
M.....	Re-read 2 O	X	X	X	X
W.....	Re-read numerals O	O	O	O	O
H.....	O	O	O	O	O
B.....	O	O	O	Re-read 1000 O	O
G.....	O	O	O	Re-read 1000 O	O

Explanation of symbols—
"X" = re-read and copied numerals, and computed from copied numerals.
"O" = computed immediately and directly from the problem card without re-reading.

It is important at the beginning of the discussion of the two methods of procedure that the most essential difference between them be set forth clearly. The difference lies primarily in the number of mental steps involved in the two methods. At least two additional mental steps are required by the method of computation from copied figures, namely, re-reading of numerals and copying them on the computation card. The method of computation direct from the problem card avoids

these two steps. A large pedagogical significance attaches to the elimination of two such mental steps. By their elimination it would appear that many opportunities for error are avoided and valuable economies of time and of mental and physical effort may be effected.

The description of the procedures which were used in computation that follows, is concerned only with such procedures as were exhibited by the subjects who computed directly from the problem cards. It was found impossible to interpret precisely the records of eye-movements over copied figures in respect to the location of the pauses on the figures. The processes of computation could be followed to the solution of the problem in only a limited number of the records even of those subjects who used the method of direct computation.

The procedure of direct computation from the problem card appears to have been followed without regard to the quality of first reading which the numerals had received. Cases of direct computation are found as sequels both to partial first readings and to whole first readings of the numerals. In Plate XI direct computation is observed to have followed upon whole first readings of both numerals of the problem. Similar illustrations are found in plates VIII and X.

In Plate XII, however, the two numerals had been only partially read during the first reading. In this instance the numerals never were read completely at any reading. Similar cases were found with other subjects. Evidently for some subjects only a partial reading of the numerals of a problem is necessary for the successful use of such numerals in computation.

It is important to notice in this connection that the same subjects do not always use the same procedure in computing with different sets of problems under different conditions. Illustrations are found in the cases of subjects G and H. These two subjects used the method of direct computation with the "five problems," as reported immediately above. With the "seven problems" of the first preliminary study, however, they followed the procedure of computation from copied figures.

An explanation of these facts should begin with the very probable premise that these two subjects, who were adults and advanced graduate students, were able to follow successfully either procedure in solving such simple arithmetical problems as were used in this investigation. The method of direct computation was followed with greater facility with the "five problems" than with the problems of the first preliminary study because of the greater obviousness of the answers to the "five

problems." The numerals of most of these problems had been so selected as to make the answers even numbers, or numbers with every digit the same, in order to minimize the labor of computation. It is probable also that the subjects worked at somewhat higher tensions when seated before the camera for solving the "five problems" than when seated at an ordinary desk for solving the "seven problems." Such higher tensions might well have influenced the workers to select the quicker direct method rather than the slower copying method of solving.

During the process of computation, the numerals, which are in the context of the problem and with which the computation is concerned, do not receive the same quality of attention from the subjects. This fact is most clearly in evidence when two numerals appear in the context of a problem. The records show that more pauses and pauses of greater average duration were located on the digits of one of the numerals than on the digits of the other numeral. In the cases of shorter numerals, pauses were located upon only one of the numerals, while the other numeral was retained in memory. By such unequal distribution of attention one of the numerals was, in effect, made the "base of operations" during the computation. Three examples of this procedure were found in the solving of the problems in which the shorter numerals appeared. The records appear in detail in plates III, VIII, and IX. In Plate IX, which will serve as an illustration, the numeral 357 was used as the "base of operations," while the numeral 1643 was held in memory.

An example of the same procedure with the longer numerals of Problem 3 is found in Plate XII. In this instance the numeral 243,987 was used as the "base of operations." Five pauses with an average pause-duration of 38/50 of a second were located on this numeral during the process of computation. On the other hand, on the numeral 21,765 only four pauses were located during the computation, and their average duration was only 28.75/50 of a second.

Two further examples of the procedure are found in the solution of Problem 5 by subjects H and G in plates XIII and XV. Interpretation of plates XIII and XV is complicated to a large extent by the presence in the records of a number of pauses which were not used strictly in the processes of computation. Such additional pauses were used apparently in locating the digits next in order for computation, or else they were used in directing the hand when it was engaged in recording figures of the answer. Such pauses, therefore, may be referred to as locating- and as recording-pauses respectively.

TABLE XXVI

ANALYSIS OF THE PROCESS OF COMPUTATION IN WHICH ONE NUMERAL IS USED AS THE "BASE OF OPERATIONS"

(Time unit = 1/50 of a second)

Plate in which the record of the computation appears	XIII		XV	
Numerals which appear in the context of Problem 5 and with which the computation is concerned	1,918,564 ("Base of operations")	617,453	1,918,564 ("Base of operations")	617,453
Ordinal numbers of locating- and recording-pauses, as found on plate	38, 41, 48, 52, 53, 57	43, 47	28, 31, 34, 39, 43, 44, 45, 46	23, 26, 41
Ordinal numbers of computation pauses, as found on plate	39, 42, 45, 49, 54, 55, 58	40, 44, 46, 50, 51, 56	25, 29, 32, 35, 37, 40, 42	24, 27, 30, 33, 36, 38
Respective durations of computation pauses, as found on plate	11, 13, 24, 13, 30, 13, 36	23, 20, 17, 12, 20, 13	29, 10, 19, 20, 26, 26, 32	10, 9, 18, 26, 22, 23
Sum of the durations of computation pauses	140	105	162	108
Total number of computation pauses	7	6	7	6
Average duration of computation pauses	20.0	17.5	23.1	16.0

NOTE.—Pauses were distinguished as computation pauses, and as locating- and recording-pauses, upon the basis of the judgment of the author.

It was found impossible to distinguish the locating- and recording-pauses in plates XIII and XV from the computation pauses with absolute certainty. An effort was made, however, after a detailed study of the original reports of subjects H and G on their solutions of Problem 5, to separate the locating- and recording-pauses from the computation pauses. The result appears in Table XXVI. An inspection of the table shows that both subjects H and G used the numeral 1,918,564 as the "base of operations" during computation.

The longer numeral in five of the six cases, which are described above, was taken as the "base of operation." Such selection of the longer numeral is probably in keeping with the practice common in the solving of arithmetical problems which, when two numerals are arranged for computation, places the greater numeral first in order and relates the second numeral to the greater. The larger number of pauses upon the longer numeral, which at the same time is the "base of operations," is due to the fact that computation both begins and ends with the longer numeral, and to the further fact that in the longer numeral an additional digit appears for reading.

The large difference between the average duration of the pauses on the numeral which was used as the "base of operations" and the average duration of the pauses on the other numeral is significant of a difference in function between the two sets of pauses. The pauses on both numerals necessarily must use such time as is sufficient for recognition on the part of the reader of the digits with which the pauses are severally concerned. In addition to the work of recognitions, however, some of the pauses on the "base of operations" numeral evidently perform service in the more strictly arithmetical processes. For such service a greater pause-duration would undoubtedly be necessary.

3. SUMMARY

1. Two types of re-reading of numerals are distinguishable. Simple re-reading is concerned with verification of details of the numerals. Re-reading for copying is concerned with reading the numerals for copying on the computation card.

2. Two of the subjects normally re-read all of the numerals. The four other subjects normally do not re-read the numerals. Whether the numerals are or are not re-read depends upon the habits of individual subjects.

3. Two methods of proceeding with the numerals after the first reading are distinguishable. In the one, computation begins immedi-

ately and is carried on directly from the context of the problem. In the other case the numerals are re-read and copied, and the computation proceeds from the copied figures. By the former method, two mental steps are saved.

4. The method of immediate computation direct from the context of the problem is used without regard to whether the numerals have received a partial or a whole first reading.

5. During computation one numeral is taken as the "base of operations." A large number of pauses and pauses of greater average duration are located on the digits of this numeral than on the digits of the other numeral. The significance of the greater duration of such pauses probably lies in the additional work of the more strictly arithmetical processes which seems to have been done during these pauses.

CHAPTER IX

THE READING OF ISOLATED NUMERALS IN LINES

1. INTRODUCTION

As was stated in the introductory paragraphs of this report such attention as has been given to the reading of numerals in previous experiments in the psychology of reading has been incidental to other purposes. The numerals which were read in previous experiments were in each case isolated numerals in lines. Gray,[1] when investigating the perception span of good and poor readers, had a number of individuals read short lines of unspaced digits and groups of the same digit as well as selections of words with meaning. A summarizing paragraph at the end of his discussion contains the conclusion that differences between the span of attention of the good and poor reader disappear in a very large measure when digits or groups of the same digit are read.

Dearborn,[2] while interested chiefly in the span of attention and in the question as to whether perception proceeds by number wholes or by individual digits, had several subjects read lines of digits which were printed consecutively without spacing, and lines of numerals varying in length from two to six digits. In the records which were obtained from these readings he observed that the time required for reading the same number of unspaced digits in a line was greater when the subjects grouped the digits by fours than when they grouped the digits by threes. He noticed also that the time required for reading numerals increased with increases in the digit-lengths of the numerals. Attention was called, at the same time, to the larger number of "shifts" or "breaks" in the fixations on numerals than in the fixations on words, and the opinion was expressed that a single digit was probably sometimes the unit of perception in the two-digit numerals.

In a preliminary study of the present investigation, data were presented concerning the reading of numerals arranged in columns.

[1] C. T. Gray, "Types of Reading Ability as Exhibited through Tests and Laboratory Experiments," *Supplementary Educational Monographs*, Vol. I, No. 5 (1917), p. 146.

[2] W. F. Dearborn, "Psychology of Reading: An Experimental Study of the Reading Pauses and Movements of the Eye," *Columbia University Contributions to Philosophy and Psychology*, Vol. XIV, No. 1. New York: The Science Press, 1906. See chapter x, "The Number Span of Attention," pp. 67–73.

In that study special attention was called to the fact that the digits of the numerals were read in groups. The purpose of the present study is to examine in detail the readings of a representative number of numerals of each of the several digit-lengths of from one to seven digits. Variations in the readings of the numerals of the several lengths are reported and the reading habits, which were exhibited by individual readers, are described. The records of the movements of the eyes of the subjects while engaged in reading numerals are given in plates XVI–XXV, and the data from the records are condensed and arranged in tables XXVII–XXXI.

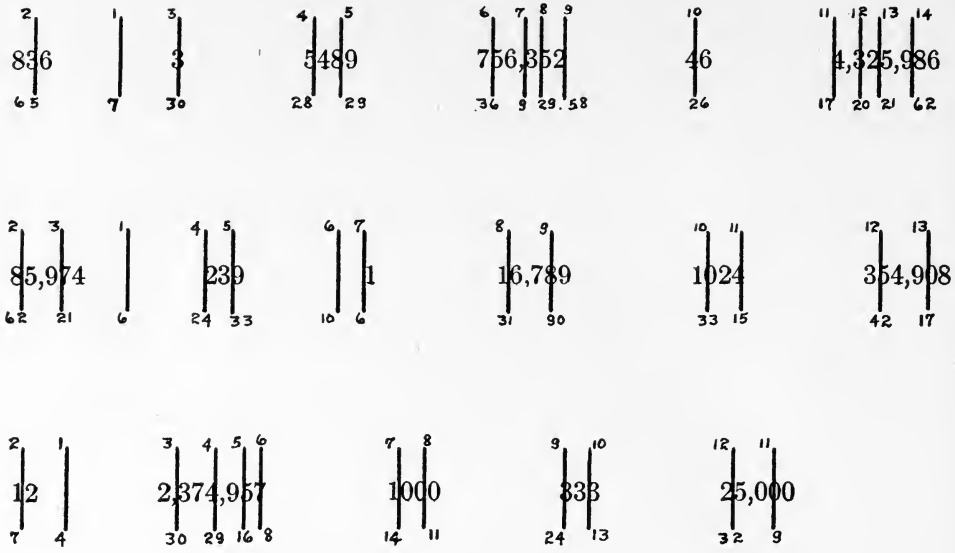
In the plates the variations in length of the vertical lines above and below the lines of printed numerals were provided merely for convenience in drawing in the numbers of the various pauses.

When the initial pause of a line did not fall on one of the digits of the first numeral in the line, such a pause was not included in the tables. When an initial pause fell on the first numeral of a line and was followed by a regressive movement, such a pause was not counted in the tables. It is obvious that counting pauses of the latter sort would have given in each case an additional pause to the first numeral in the line merely because of its position as first numeral in the line.

In plates XVI and XVII the reading of isolated numerals by Subject G is represented. At the beginning of each line of numerals an initial regressive movement was found necessary in the effort to locate the first digits of the first numeral. Subject G reads with relatively few pauses, but with pauses of relatively long duration. The pauses vary widely in duration; the range of variation extends from 4/50 to 90/50 seconds with the average duration at 33.88/50 of a second. Single pauses, when they are located on the longer numerals, tend to perceive two or three digits rather than one or two. In three instances the subject accomplished the remarkable feat of reading four digits during a single pause. The three instances are found in Plate XVII; two are in the first line with the numerals 9317 and 5,236,795; and the third is in the second line with the numeral 1928.

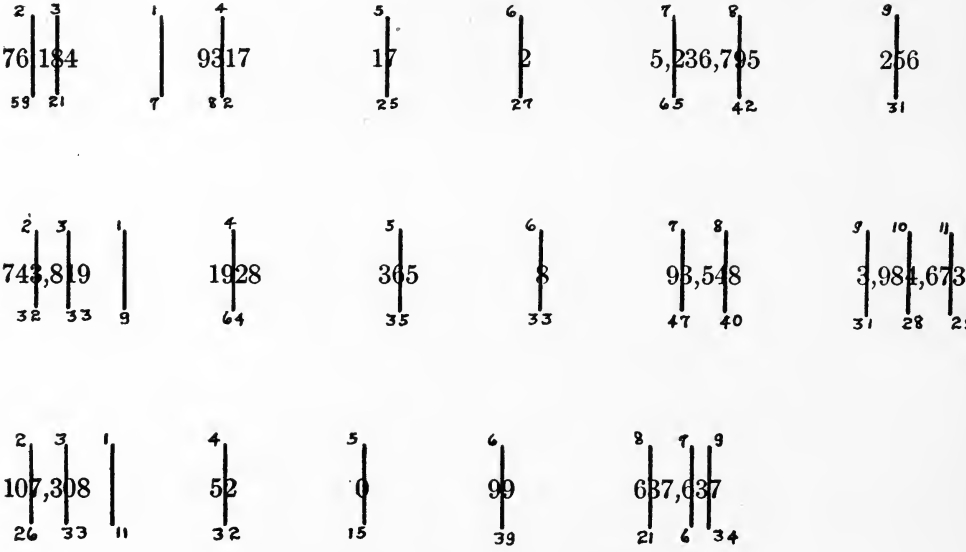
In plates XVIII and XIX appear the readings of isolated numerals by Subject H. This subject read with a relatively large number of pauses, but with pauses of relatively short duration. The pauses varied in duration from 4/50 to 56/50 seconds. The average duration, which was 19.32/50 of a second, was shorter than that of any other subject. Many short guiding-pauses appear. Single pauses, even when they are located on the longer numerals, tend to include only

PLATE XVI



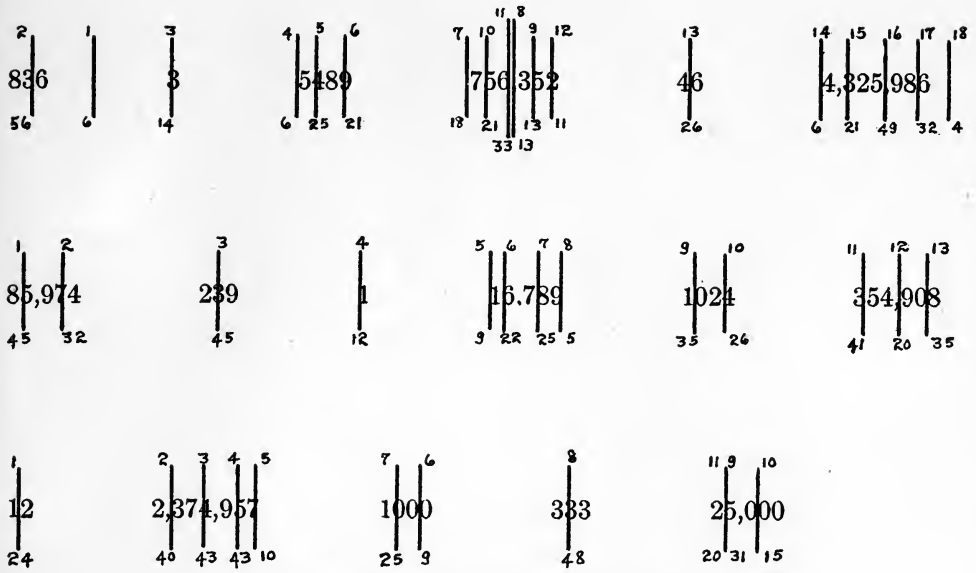
Reading of isolated numerals by Subject G

PLATE XVII



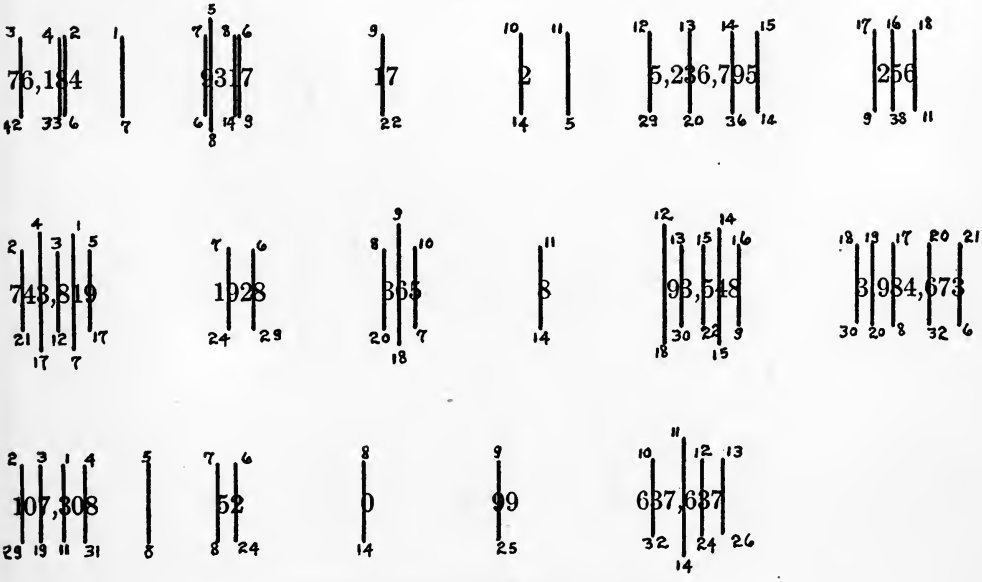
Reading of isolated numerals by Subject G

PLATE XVIII



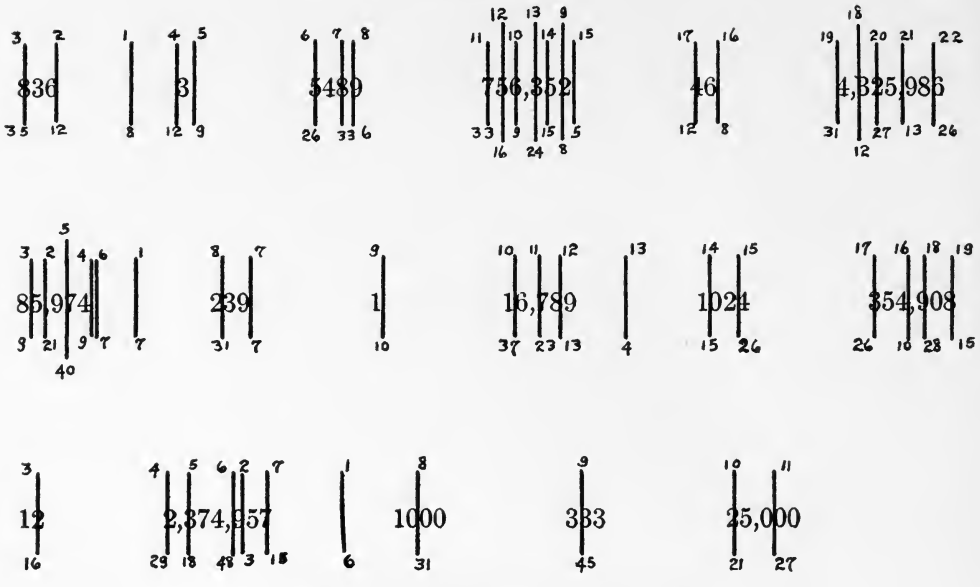
Reading of isolated numerals by Subject H

PLATE XIX



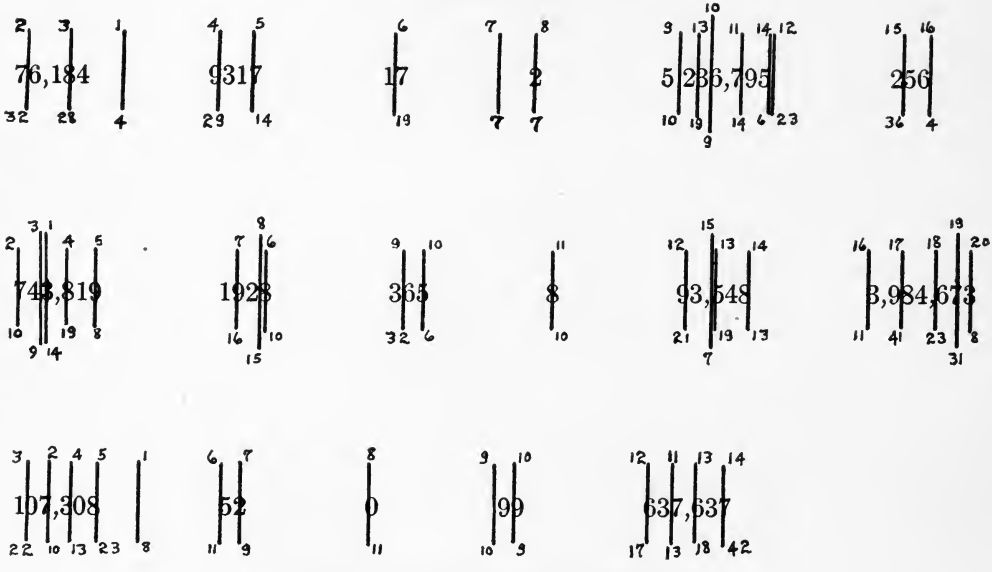
Reading of isolated numerals by Subject H

PLATE XX



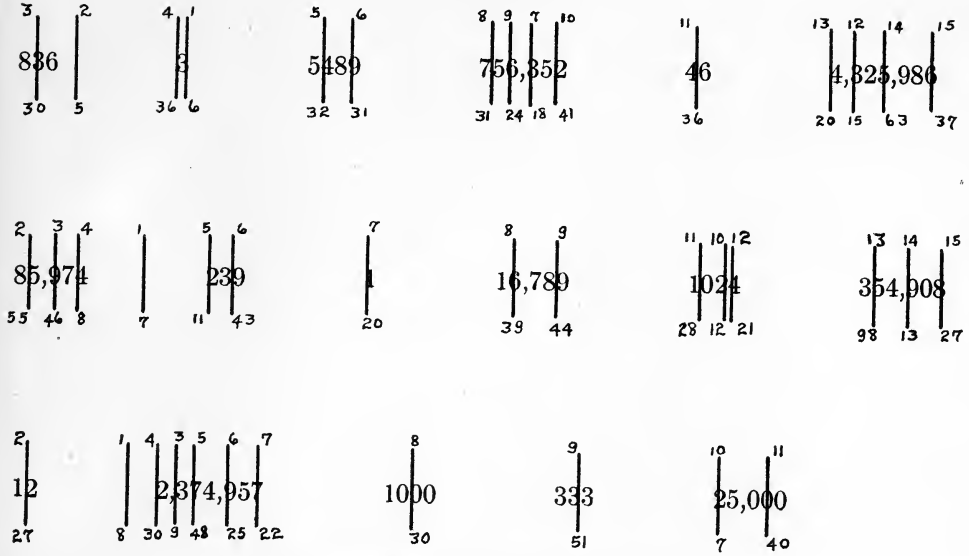
Reading of isolated numerals by Subject M

PLATE XXI



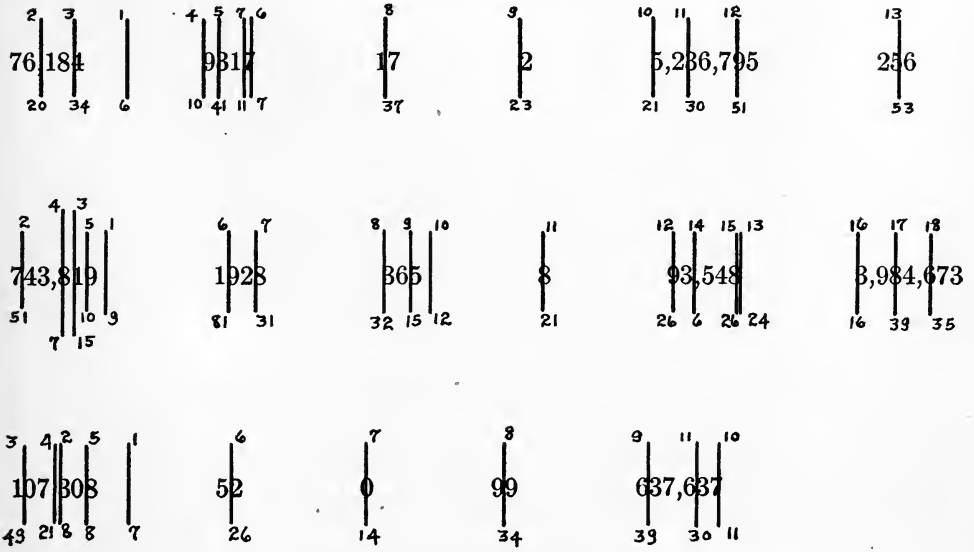
Reading of isolated numerals by Subject M

PLATE XXII



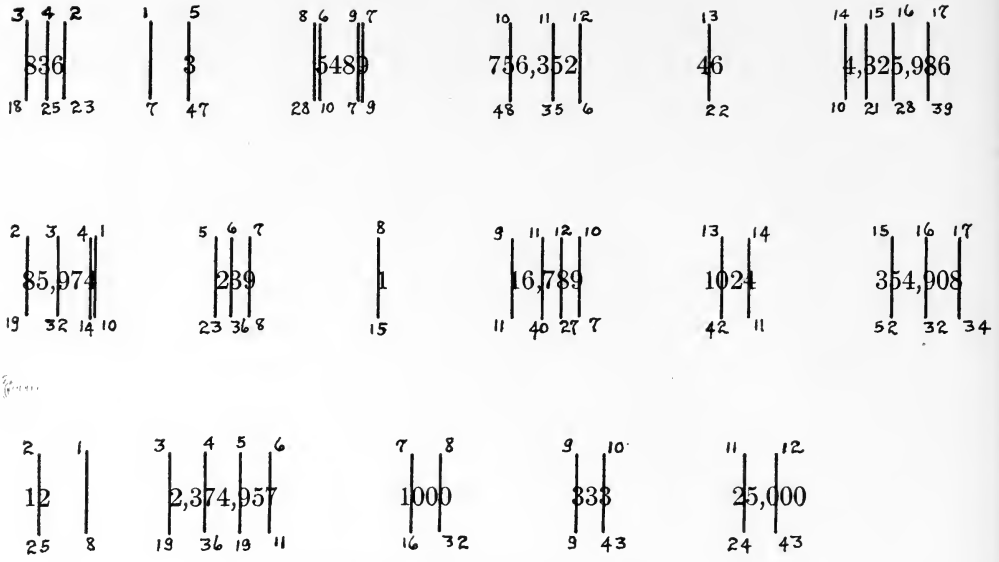
Reading of isolated numerals by Subject B

PLATE XXIII



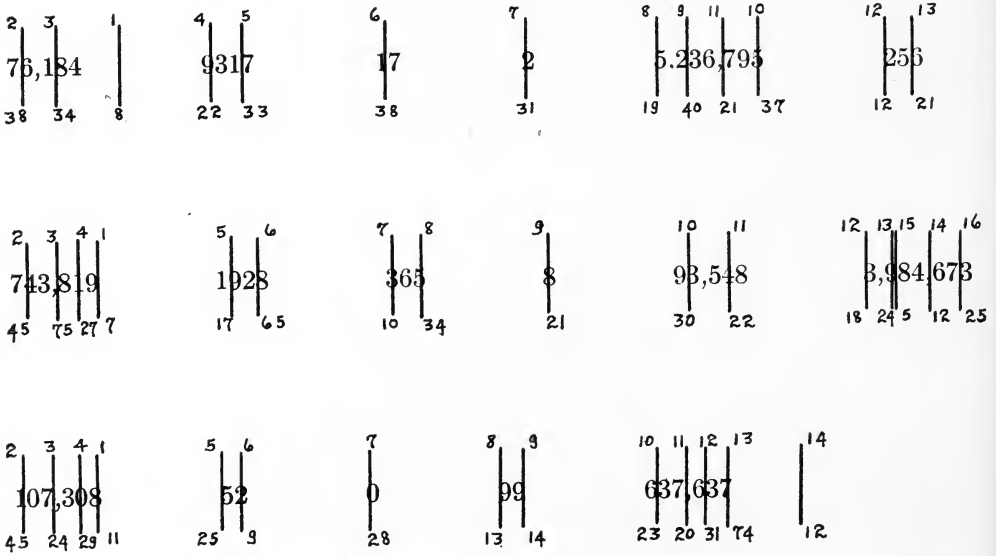
Reading of isolated numerals by Subject B

PLATE XXIV



Reading of isolated numerals by Subject W

PLATE XXV



Reading of isolated numerals by Subject W

one or two digits at a pause. The readings, which were given the numerals 756,352 in Plate XVIII and 93,548 in Plate XIX, are illustrations of very detailed reading. The readings given the last three numerals on Plate XVIII, the special numerals 1000, 333, and 25,000 show fewer pauses than H gave to other numerals of like lengths. He evidently read 1000 as a whole. A relatively large number of regressive pauses is found on these plates.

The readings of Subject M are described in plates XX and XXI. His methods of reading the numerals were similar to those of Subject H, to which attention was called in connection with plates XVIII and XIX. The total reading-time for all the numerals was less for Subject M than for any other subject, despite the relatively large number of pauses which he used. Several instances appear in these plates of short initial and final pauses on the same longer numerals. The readings of the numerals 4,325,986 and 16,789 in Plate XX and of 5,236,795 and 743,819 in Plate XXI illustrate such use of the guiding-pauses. The special numerals were read in the same manner as other numerals of like lengths. Plates XXII and XXIII record the reading of isolated numerals by Subject B. It appears that this subject's readings are irregular in respect to number of pauses on numerals of the greater lengths. Some of the longer numerals were read with few pauses. The numerals 5,236,795 and 3,984,673 in Plate XXIII are illustrations of this type, while the numerals 9317 and 743,819 on the same plate were read with a comparatively large number of pauses. The numeral 1000 was evidently read as a whole. Initial regressions appear consistently in each line.

The readings of isolated numerals by Subject W are presented in plates XXIV and XXV. A persistent use of two pauses appears in the readings of numerals of from three- to five-digit lengths. Two instances are seen in the numerals 5489 and 16,789 in Plate XXIV when even the re-readings of the numerals were done with pairs of pauses. Initial regressions occur consistently.

2. TWO TYPES OF PAUSES

When a detailed examination is made of the pauses with which the numerals were read, it appears that they represent two distinct types.

The two types are distinguished by differences in function. Pauses of the first type, which may be called strictly reading-pauses, were probably used in recognizing the identity of the digits of the numerals and the relations between the digits. Such pauses are invariably located on the numerals. Their durations are approximately equal to, or greater

than, the average duration of the pauses of the subject whose records are under consideration. A preponderant number of the pauses of any subject are of this first type.

Pauses of the second type, which may be called guiding-pauses, were probably used in locating the first digits or the last digits of the numerals. They are found on the initial or final digits of numerals, and more frequently on numerals of greater digit-lengths. Some of these pauses appear on the lines between the numerals. The first pause in any line was almost invariably of this type and of very brief duration when compared with other pauses. Subjects H and M used larger numbers of pauses of this type than any of the other subjects.

3. DIFFERENCES IN THE READINGS OF NUMERALS OF DIFFERENT LENGTHS

Upon inspection of the last row of Table XXVII it is found that the average total reading-time per numeral increases steadily from an average of 21.45/50 of a second for the one-digit numerals to an average of 104.54/50 of a second for the seven-digit numerals. The same continual increase is found almost without exception in the rows of the several subjects. Likewise the average number of pauses per numeral, when the records of all subjects are averaged, increases steadily from the average of 1.15 pauses on one-digit numerals to the average of 4.15 pauses on seven-digit numerals. It is clear therefore that the total reading-times and the number of pauses which were required to read a numeral, depended upon its digit-length.

The average duration of the pauses, on the other hand, does not depend upon the length of the several numerals in the same consistent fashion as does the average number of pauses per numeral. The details may be found in Table XXVIII, where it is seen that both in the rows for individual subjects as well as in the row which presents averages for all of the subjects, the average pause-duration not only fails to increase steadily with increasing lengths of the numerals but in several cases actually decreases.

With three of the subjects, M, H, and G, on the other hand, the average duration of the pauses increases steadily from that of the numerals of one digit to that of the numerals of three digits. Two of these three subjects, M and G, however, as may be observed in Table XXIX, read the one, two, and three digits of the one-, two-, and three-digit numerals, respectively, for the most part with a single pause. In the case of Subject B, also, a steady increase in pause-duration is found when the one-, two-, and three-digit numerals were read at single

TABLE XXVII

AVERAGE NUMBER OF PAUSES AND AVERAGE READING-TIME PER NUMERAL FOR ISOLATED NUMERALS

(Time unit = 1/50 of a second)

Subject	Average per Numeral	Digit-Length of Numerals						
		1	2	3	4	5	6	7
M	Number of pauses	1.25	1.25	1.5	2.5	3.0	4.5	5.0
	Reading-time	12.5	22.25	44.75	49.25	64.5	83.25	110.75
H	Number of pauses	1.25	1.5	2.5	2.75	4.0	4.75	4.5
	Reading-time	14.75	25.0	47.0	49.0	81.0	90.25	103.5
G	Number of pauses	1.25	1.0	1.25	1.5	2.0	2.5	3.25
	Reading-time	26.5	25.0	47.0	62.75	92.75	78.5	99.7
W	Number of pauses	1.0	1.25	2.5	2.5	2.75	3.0	4.25
	Reading-time	28.5	39.75	52.5	61.0	68.5	113.0	93.5
B	Number of pauses	1.0	1.0	1.75	2.75	2.75	3.75	3.75
	Reading-time	25.0	31.5	49.0	76.25	82.0	105.25	115.25
All Subjects	Number of pauses	1.15	1.2	1.9	2.4	2.9	3.7	4.15
	Reading-time	21.45	28.7	48.05	59.65	77.75	94.05	104.54

NOTE.—Each subject read four of each of the one-digit, two-digit, ... seven-digit numerals.

TABLE XXVIII

AVERAGE PAUSE-DURATION OF ISOLATED NUMERALS OF THE SEVERAL DIGIT-LENGTHS

(Time unit = 1/50 of a second)

Subject		Digit-Length of Numerals						
		1	2	3	4	5	6	7
M	Average pause-duration	10.0	17.8	29.8	19.7	21.5	18.3	22.2
	Average variation	2.3	6.2	16.6	7.5	9.4	6.8	12.0
	Total number of pauses	5	5	6	10	12	18	20
H	Average pause-duration	11.8	16.6	18.8	17.8	20.3	19.0	23.0
	Average variation	1.8	4.7	9.8	8.9	10.6	6.7	9.0
	Total number of pauses	5	6	10	11	16	19	18
G	Average pause-duration	21.2	25.0	37.6	41.8	46.4	31.4	30.6
	Average variation	9.4	3.8	11.8	20.8	18.1	8.9	11.5
	Total number of pauses	5	4	5	6	8	10	13
W	Average pause-duration	28.5	23.8	21.0	24.4	24.9	37.6	22.6
	Average variation	10.5	6.6	9.0	14.0	9.2	12.8	7.0
	Total number of pauses	4	5	10	10	11	12	17
B	Average pause-duration	25.0	31.5	28.0	27.7	29.8	28.1	30.7
	Average variation	5.3	5.0	13.1	14.1	12.5	17.2	11.9
	Total number of pauses	4	4	7	11	11	15	15
All Subjects	Average pause-duration	19.3	22.9	27.0	26.3	28.4	26.9	25.8

NOTE.—Each subject read four each of the one-digit, two-digit, ... seven-digit numerals.

TABLE XXIX

NUMBER OF ISOLATED NUMERALS OF THE SEVERAL DIGIT-LENGTHS WHICH WERE READ WITH VARIOUS NUMBERS OF PAUSES PER NUMERAL

Digit-length of numerals	1		2		3			4				5					6							7						
Various numbers of pauses per numeral with which the numerals were read	1	2	1	2	1	2	3	1	2	3	4	1	2	3	4	5	1	2	3	4	5	6	7	1	2	3	4	5	6	7
Subject M	3	1	3	1	2	2	..	..	2	2	..	..	2	..	2	..	..	..	1	1	1	1	..	..	..	..	1	2	1	..
Subject H	3	1	2	2	..	2	2	..	2	1	1	..	..	2	..	2	..	..	..	3	..	..	1	..	..	..	2	2	..	..
Subject G	3	1	4	..	3	1	..	2	2	..	..	..	4	..	..	..	..	3	..	1	..	..	..	..	1	1	2	..	..	..
Subject W	4	..	3	1	..	2	2	..	3	..	1	..	2	1	1	..	..	..	4	..	..	..	..	..	..	..	3	1	..	..
Subject B	4	..	4	..	2	1	1	..	2	1	1	..	2	1	1	..	..	..	1	3	..	..	..	..	..	..	2	1	1	..
Total for all subjects	17	3	16	4	7	8	5	2	11	4	3	..	10	4	4	2	..	3	6	8	1	1	1	..	1	3	9	6	1	..

NOTE.—Each subject read four each of the one-digit, two-digit, ... seven-digit numerals.

pauses. It is evident, therefore, that with certain subjects and to this limited extent, the pause-duration does increase with increases in the number of digits read.

Numerals of the same length exhibited a notable consistency in the number of pauses with which they were read by the several subjects. As may be observed in Table XXIX two different numbers of pauses include the number of pauses used in reading a preponderant number of the numerals of each length. The one- and two-digit numerals were read for the most part by single pauses. One and two pauses were used with the three-digit numerals and two and three pauses with the four- and five-digit numerals in most cases. The longer numerals exhibit more variety in the number of pauses with which they were read. With these numerals, the habits of individuals seem larger deciding factors than the tendency to a conventional number of pauses.

4. THE SPECIAL NUMERALS

The six special numerals, 0, 99, 333, 1000, 25,000, and 637,637, were included among the other numerals, in order that data might be obtained concerning such variations in the readings of numerals, as might be due to familiarity and to regularity in form on the part of the numerals read. The data for the two special numerals 1000 and 25,000 show consistent and significant variations from the data which represent the readings of ordinary numerals of like lengths. As may be observed in Table XXX the two numerals 1000 and 25,000 exhibit markedly shorter average reading-time per numeral, average number of pauses per numeral, and average pause-durations, than the ordinary numerals of the same lengths with which they are compared.

TABLE XXX

READING OF SPECIAL NUMERALS COMPARED WITH READING OF OTHER ISOLATED NUMERALS OF CORRESPONDING DIGIT-LENGTHS

(Time unit = 1/50 of a second)

Special numerals	0	99	333	1000	25,000	637,637
Corresponding digit-lengths of other isolated numerals with which the special numerals are compared	1	2	3	4	5	6
Average reading-time per numeral:						
Special numerals	16.4	30.8	46.6	33.6	53.8	93.0
Other numerals	21.45	28.7	48.05	59.65	77.75	94.05
Average number of pauses per numeral:						
Special numerals	1.0	1.40	1.40	1.6	2.6	4.50
Other numerals	1.15	1.2	1.9	2.4	2.9	3.7
Average pause-duration:						
Special numerals	16.4	22.0	33.29	21.0	24.46	25.82
Other numerals	19.3	20.86	27.04	26.28	28.43	26.88

NOTE.—Each subject read each special numeral once, and at the same reading with the other isolated numerals.

Each of the two numerals represents the special quality of regularity of form and the quality of familiarity which were described in the discussion of the numeral 1000 in chapter vii. Regularity of form, in what may be called common fashion as distinguished from the special fashion of regularity which has been described for the numeral 1000, is a quality of the other special numerals 99, 333, and 637,637. Such regularity of form, however, did not seem to be significant enough to affect important variations in the readings of those numerals. This immediate fact and the facts which are noted above appear in re-enforcement of the conclusion reached regarding the numeral 1000 in chapter vii. Evidently the qualities of special regularity of form and familiarity are influential factors in the reading of numerals, even when the numerals are isolated in lines.

5. TWO METHODS OF ATTACK IN READING NUMERALS

Two distinct methods of attack were used by different subjects in reading the numerals. By one method a relatively large number of pauses of relatively short average duration were used in the readings, and by the other relatively few pauses of relatively long average duration were used. The contrast between the two methods is sharply outlined by differences in the data for subjects H and G, who represent the first and second methods respectively, as such data are arranged in Table XXXI. Subject M also read by the first method and subjects W and B by the second method. In plates XVIII and XIX a relatively large number of guiding-pauses are found, while comparatively few appear in plates XVI and XVII. The shortest total reading-times for the whole

TABLE XXXI

SPEED AND THE TWO METHODS OF ATTACK USED IN READING ISOLATED NUMERALS

(Time unit = 1/50 of a second)

Methods of attack	Many Pauses of Short Average Duration		Fewer Pauses of Longer Average Duration		
Subjects	M	H	G	W	B
Total reading-time for all numerals	1549	1642	1728	1797	1937
Total number of pauses used in reading all numerals	76	85	51	69	67
Average pause-duration	20.38	19.32	33.38	26.04	28.92

NOTE.—Each subject read four each of the one-, two-, three-, ... and seven-digit numerals or a total of twenty-eight numerals.

set of isolated numerals are found in the records of subjects M and H who used the many-short-pauses method of attack in their readings.

The number of digits which are read at a single pause depends upon the method of attack which the subject uses, and upon the number of digits which a numeral offers for reading. Subjects M and H, who use the method of many short pauses, tend to perceive one and two digits at a pause. Subjects G, B, and W, on the other hand, who use the method of few and long pauses, tend to perceive two and three digits at a pause. The one and two digits of the one- and two-digit numerals were almost invariably read by single pauses by all subjects. In all of the plates which describe the readings of M, G, W, and B instances are to be found among the readings of the longer numerals in which as many as three digits were read at a pause.

In only three instances, however, when the special form numeral 1000 is not brought into consideration, as many as four digits are known to have been read at a single pause. All three of the instances are in the readings of Subject G and are reported on Plate XVII. Two instances occur on the four-digit numerals 9317 and 1928, and the other on the seven-digit numeral, 5,236,795. Evidently the reading of four digits at a single pause is very exceptional and it is significant that it occurs only in the records of the subject, who used the smallest number of pauses with the greatest average duration.

6. SUMMARY

1. Two types of pauses are distinguished by differences in duration. The strictly reading-pauses are probably used to recognize the identity of the digits of the numerals, and the relations between them. The guiding-pauses are used probably in locating the initial and final digits of numerals.

2. The average total reading-time per numeral and the average number of pauses per numeral increase gradually from the averages of the one-digit numerals to the averages of the seven-digit numerals.

3. With three subjects the average duration of pauses increased with increases in the number of digits read by the pauses.

4. Two different numbers of pauses include the number of pauses used in reading a preponderant number of the numerals of each length. The greatest consistency in this respect is found in the numerals of from one to five digits in length.

5. The quality of familiarity, rather than the quality of regularity of form reduced the average number of pauses and the total reading-

times of the numerals 1000 and 25,000 below the averages of ordinary numerals of like lengths.

6. Two distinct methods of attack are used in reading numerals. By the one method a relatively large number of pauses of relatively short average duration is used, and by the other method relatively few pauses of relatively long average duration are used. The subjects who employed the former method read the numerals in shorter total reading-time.

7. The usual number of digits read per pause is one or two, or two or three according to the habits of the subject who is reading. One subject is able to read as many as four digits at single pauses.

CHAPTER X

COMPARISONS OF RATES OF READING

1. COMPARISON OF THE SUBJECTS OF THE PRESENT INVESTIGATION WITH THE SUBJECTS OF SCHMIDT'S INVESTIGATION IN RESPECT TO RATES OF READING

Attention has been called in previous sections of this report to the fact that larger numbers of pauses of relatively greater average duration were used in reading the arithmetical problems than are commonly used in reading ordinary prose materials. It was decided, therefore, to test the reading-speeds of the several subjects of the present investigation with a different type of reading-material. The data, which were thus obtained, could then be compared with other data which represented he reading-speeds of similar individuals with similar materials.

The type of material, which was selected as most appropriate for this purpose, was that of ordinary expository prose. The text of the selection was taken from Judd's *Psychology of High-School Subjects*, a volume which was familiar in a general way to all of the readers. The data obtained with this material were to be compared with the results reported by Schmidt[1] in a previous investigation of the reading of "light passages from James's *Psychology*"[2] by 45 "adults, mostly graduate and undergraduate students."[3]

The conditions which governed the readings of the two selections were similar for the most part. In one respect, however, an important difference obtained. In Schmidt's investigation the subjects were instructed to "read rapidly for the thought."[4] In the present investigation, out of deference to purposes which are stated in the latter part of this chapter, the subjects were instructed to read for a "clear understanding" and at "normal speed." Further details concerning this material and the conditions under which it was given appear in chapter vi.

In order to facilitate the comparison certain rearrangements of the data as originally reported by Schmidt were made. The time unit used in his investigation was the sigma, or 1/1000 of a second. Figures

[1] W. A. Schmidt, "An Experimental Study in the Psychology of Reading," *Supplementary Educational Monographs*, Vol. I, No. 2 (1917), p. 42.

[2] *Ibid.*, pp. 32, 50. [3] *Ibid.*, p. 34. [4] *Ibid.*, p. 28.

which represented duration of time in this unit were converted into other figures representing the same durations in units of 1/50 of a second. By multiplying average pause-duration by average number of pauses per line figures were obtained for the average reading-time per line. In this way a single number is found to represent rates of reading. The data from the two investigations are arranged in Table XXXII.

When the data in Table XXXII are compared it is found that the subjects of the present investigation read at conspicuously higher rates, as judged by average reading-time per line, than the subjects of Schmidt's investigation. The superiority in speed on the part of the former subjects is due in largest measure to the decidedly shorter average durations of their pauses, although they also used fewer pauses per line.

TABLE XXXII

SUBJECTS OF THE PRESENT INVESTIGATION COMPARED WITH THOSE OF SCHMIDT'S INVESTIGATION IN RESPECT TO SPEED OF READING

(Time unit = 1/50 of a second)

	SUBJECTS			
	Average Number of Pauses per Line*	Average Pause-Duration	Average Deviation	Average Reading-Time per Line‡
Of the present investigation (5 adults)	6.05	10.75	2.87	65.04
Of Schmidt's investigation (45 adults)	6.50	15.41†	3.93	100.17

* The line-length of the materials read by Schmidt's subjects was 90 mm.; that of the materials read by the subjects of the present investigation was 93 mm.

† Schmidt's results, which are reported by him in time units of sigma (1/1000 of a second), are here presented in time units of 1/50 of a second.

‡ The average reading-time per line is obtained by multiplying the average number of pauses per line by the average pause-duration.

The advantage in favor of the subjects of the present investigation is emphasized by the fact that they were reading under instructions which called for only "normal speed," whereas the other subjects read under the instructions, "read rapidly for the thought."

So large a number of adult cases was included in the investigation by Schmidt that the figures reported by him may be taken as representing reliable averages of the performances of such subjects. Upon the basis of comparison with the results thus reported it is concluded that the subjects of the present investigation may be classified as decidedly better than average adults in respect to speed of reading. The implication of the foregoing paragraphs and of this conclusion is obvious. The larger number of pauses of relatively greater duration, which was

found in the readings of the problems, is due to the nature of the problems as a type of reading-material rather than to slow speeds on the part of the readers.

2. COMPARISONS OF RATES OF READING THE THREE TYPES OF READING-MATERIALS

Significant evidence of differences between types of reading-materials, and concerning the nature of the differences as well, may be secured by comparisons between the rates with which the subjects read the different types of materials. With such a purpose in view, comparisons were arranged between the data from the readings of the problems, of the isolated numerals, and of the ordinary expository prose selection. The conditions which governed the readings of all three types of materials were essentially similar in that they called for such kinds of reading as are most customary for the several types; and for "normal speed" on the part of the subjects. The data which were obtained are, therefore, representative of normal performances on the parts of the readers with these three types of materials. Table XXXIII was designed to facilitate the comparison.

TABLE XXXIII

COMPARATIVE DATA FROM READINGS OF FIVE PROBLEMS, ORDINARY PROSE, AND ISOLATED NUMERALS

(Time unit = 1/50 of a second)

	Subjects						Average for All Subjects
	G	B	M	W	H	Hb	
Average number of pauses per line on:							
The five problems (first reading)	6.66	7.25	8.41	7.25	9.83	9.08	8.08
The ordinary prose selection	4.71	7.50	5.20	6.20	6.66	.....	6.05
Isolated numerals	.....	.....	.....	.....	.....	.....	.....
Average number of letters, or digits, per pause on:							
The five problems (first reading)	6.61	6.08	5.24	6.08	4.48	4.85	5.56
The ordinary prose selection	9.61	6.03	8.69	7.38	6.75	.....	7.69
Isolated numerals	2.28	1.66	1.48	1.62	1.31	.....	1.67
Average duration of pauses on:							
The five problems (first reading)	11.28	11.95	10.93	13.44	12.11	11.58	11.88
The ordinary prose selection	11.15	10.77	8.63	12.13	11.25	.....	10.75
Isolated numerals	30.57	28.62	19.85	26.11	18.18	.....	23.80

NOTE.—Satisfactory data on the reading of ordinary prose and isolated numerals were not obtained from Subject Hb.

The average line-length of the five problems materials was 93.33 mm., of the ordinary prose selection 93.0 mm.

The conspicuous feature of Table XXXIII is the marked superiority in the speed with which the ordinary prose selection was read. The greatest differences are found between the isolated numerals and the prose selection. All of the subjects exhibit these differences, both in

respect to average number of digits or letters read per pause and in respect to average duration of pauses as well. The differences indicating greater speed with the prose are in large proportion. Such differences in speed undoubtedly reflect the great and obvious differences between materials for prose and for isolated numerals in respect to mechanical form, context, and the attitudes which they inspire in the minds of readers. Evidently isolated numerals are much more difficult as reading-material than ordinary expository prose.

The differences in rates between the five problems and the ordinary prose materials are decidedly significant although not as great in proportion as those found in the comparison just drawn. Four of the five subjects read the ordinary prose at higher rates than they had used with the five problems. The higher speeds with the prose are due for the most part to fewer pauses per line, although the durations of the pauses on the prose are somewhat shorter than those on the problems. The obvious interpretation of the differences is found in the greater difficulty of the problems as reading-materials. To a large extent, as has been shown in previous sections of this report, the greater difficulty is due to the exactions of the numerals in the problems. It is probable also that greater exertions were undergone by the subjects in their efforts to grasp accurately the terms of arithmetical problems than to learn the facts contained in a selection of ordinary, expository prose.

3. SUMMARY

The following conclusions may be drawn from the discussion of this chapter: (1) The subjects of the present investigation read at decidedly higher rates than the subjects of Schmidt's investigation. They are, therefore, classified as decidedly better than average adults in respect to speed of reading. (2) The ordinary, expository prose material was read at significantly higher rates than either the arithmetical problems or the isolated numerals materials. The conclusion was drawn, therefore, that arithmetical problems and isolated numerals are decidedly more difficult as types of reading-materials than ordinary expository prose.

CHAPTER XI

PRACTICAL APPLICATION TO CLASSROOM TEACHING

1. THE QUESTION OF READING IN ARITHMETIC

Arithmetic is commonly reputed to be one of the most difficult subjects in the curriculum of the elementary school. The meagerness of the results which are obtained at the cost of such large amounts of time as are devoted to the study of the subject is a matter of continuous complaint. The changes which have come about as a consequence of efforts to improve the situation have resulted for the most part in a new selection of subject-matter for textbooks and in rearrangements of the sequence of topics. The problem exercises have become more practical in character and there is evidence of a tendency to employ in problems only such classes and magnitudes of numerals as are used in everyday affairs.

When the work which has been done in these directions is duly recognized, the significant fact remains that some of the most important divisions of the field have not yet been occupied. The number of scientific studies in the psychology of arithmetic is still surprisingly small. As compared with reading, arithmetic has been seriously neglected. Fortunately, however, much of the information which has been made available in reports on reading can be used in the study of arithmetic. A number of recent reports of experiments in the teaching of reading has served to emphasize the existence of numerous distinct types of reading materials each of which calls for the development of an appropriate type of reading ability on the part of the student. Arithmetical problems and isolated numerals as well, when considered as reading situations, represent distinct types of reading materials. The special types of procedure which adult subjects used in reading such materials have been described in previous chapters of this report.

Whatever the nature of the materials, however, reading is essentially the process of getting meaning from the printed page. When the subject-matter is difficult, patient and systematic search for the thought is necessary. Instead of taking pains in this fashion, children frequently resort to mechanical pronunciation of the words or mere scanning of the lines.[1] In such cases no adequate idea of the meaning is obtained, and

[1] Estaline Wilson, "Improving the Ability to Read Arithmetic Problems," *Elementary School Journal*, Vol. XXII, No. 5 (January, 1922), pp. 380–86.

if the materials read be arithmetical problems, a correct solution is impossible. It is scarcely an exaggeration to say that few children, including those with the experience of the upper grades, have developed a method of reading problems which could be described as a rapid and skilful attack on the reading situation.

The extent to which the appearance of numerals along with words in the text of a problem is responsible for this condition is but slightly understood. Numerals as a distinct and significant object of study have received very little attention in the literature of arithmetic. Occasional reference is made to the abstract character of the numerical concept and the difficulty of teaching children the meaning of the symbols. Discussions of classroom experience also are reported occasionally. Of these the question: What magnitudes of numerals should be used when new problems are introduced? is fairly illustrative. Numerals in the context of problems, but for exceptions of this character, have been almost completely neglected.

2. PRELIMINARY ANALYTICAL READING OF PROBLEMS

The presence of numerals among words, however, is only one of the features which distinguish arithmetical problems as a specialized type of reading materials. The existence of other equally complicating features is implied when teachers express the opinion that it is more difficult to teach the interpretation of problems than the mechanical skills of computation. Despite general acceptance of this opinion, scientific studies in the psychology of arithmetic have been concerned almost entirely with the field of operations. As a consequence, exact knowledge of the nature of the processes of interpretation is lacking for the most part.

Efforts have been made, notwithstanding, upon the basis of common observation to describe the reading situation which a problem offers and the preliminary thinking about it which is necessary before the reader is prepared for the work of computation. The most essential characteristic of a problem is the fact that it presents a series of conditions which describe a certain state of affairs. Some of the conditions appear in precise quantity. The quantities stand in definite relationships with each other and are stated in abstract terms.

Each of the elements of this complex situation must be comprehended by the student during the preliminary reading. He must draw in his imagination an accurate picture of the situation, which is described, and take account of each of the facts of relation. Following this, comes

a canvass of the plans for solving which suggest themselves, and the passing of judgment on the appropriateness of each plan to the conditions of the problem. The reading processes which are carried on in this manner are analytical in character and call for a high degree of skill and patience, as well as for a certain amount of practical acquaintance with the facts described. It is doubtful if teachers generally have acquired anything like an adequate appreciation of the confusion which children feel when confronted with a situation to be studied in this fashion.

With the intention of facilitating the formation of working habits which will bring relief to this confusion, authors of teaching manuals undertake to outline the steps which should be followed in reading and solving a problem. The first two or three steps usually provide for the preliminary analytical reading. The following directions, which appear in a recent manual as the first three of five steps, will serve as an illustration: "*First*, the pupil must read the problem and understand what it means. *Second*, he must state in his own words what the problem calls for. . . . . *Third*, he must find the material that the problem gives to work with."[1]

The reader of this report will notice immediately that no suggestions are given concerning the proper treatment of numerals. As far as the first three steps are concerned it would seem that the directions were designed for use in connection with that special type of problem only in which numerals are not included. The author appears to assume that children are able to make as rapid and prompt a disposition of the numerals as the adult subjects of the present investigation. It is clear that a rapid and thoughtful reading of problems containing numerals is not practicable under the directions as given above, unless the reader is relieved of the details of the numerals and is free to devote his entire attention to the conditions of the problem. This desirable result, as has been shown in a previous chapter of this report, can be obtained by employing the partial method of reading numerals.

3. APPLICATION OF PARTIAL READING

The importance of conducting the difficult processes of analytical reading in the most direct and unhampered manner is great enough to warrant the inclusion with the directions quoted above of any additional directions that may be necessary. Undoubtedly supplementary directions which prescribed partial reading of the numerals would greatly facilitate the preliminary reading of problems. Such a prescription

[1] Kendall and Mirick, *How to Teach the Fundamental Subjects*, p. 169. Boston: Houghton Mifflin Co.

would also serve to emphasize the significance of partial reading, and the extensive possibilities of improvement which lie in its use could begin to be realized. At this point, the reader should be reminded of the fact that the problems which were employed in the present investigation were both brief and simple. Since the partial method facilitated the reading of such problems, its value for the larger and more involved problems which are found in the ordinary textbook would be correspondingly greater. And likewise, a relief measure which was desirable for adult graduate students should prove all the more helpful to children in the elementary school.

An effort has been made, therefore, to prepare recommendations concerning the reading of problems which should be considered as additional to the directions that are quoted above. The recommendations are as follows:

1. Pupils should be taught to distinguish between the first reading and the re-reading phases in their attack on problems.

2. They should learn to consider numerals and the accompanying descriptive conditions as different elements of a problem and separable for reading purposes.

3. During the first reading, they should devote their entire attention to the conditions of the problem.

4. At the same time skill should be developed in partial reading of numerals.

5. While this skill is being acquired, pupils should be apprised of the essential similarity between the conditions of the problem and such details of the numerals as are perceived by partial reading.

Although the preceding recommendations were derived for the most part from the findings of this report, their validity does not rest on this basis alone. They derive additional support by comparison with the findings of other investigations in the field of reading. Gray, in summarizing the principles of method which were deduced from recent studies of reading, states that emphasis of the elements on which meaning depends, improves comprehension.[1] A closely related principle is stated by Freeman: "Rate of reading is increased by attending to the meaning as distinguished from the mechanics."[2] In the case of arithmetical problems, the elements referred to by Gray, obviously, are the

[1] W. S. Gray, "Principles of Method in Teaching Reading, as Derived from Scientific Investigation," *Eighteenth Yearbook of the National Society for the Study of Education*, Part II, p. 42.

[2] F. N. Freeman, *The Psychology of the Common Branches*, p. 93. Houghton Mifflin Co.

conditions of the problem and such details of the numerals as identity of the first digit and the number of digits. Attention to these items and to these alone is obtained by the use of partial reading as recommended above. The remaining details of the numerals are of the nature of "mechanics," as described by Freeman, and when the partial method of reading is used, the attention of the student is relieved of the "mechanics" and is free to search out the meaning.

Further comparisons with the results of other investigations emphasize the fact that the use of partial reading as recommended above represents a more progressive type of reading. Progress in reading, according to Freeman, consists in a decrease of the number of pauses of the eye and an increase in the scope of recognition at each pause.[1] Gray concludes that "regular rhythmical movements of the eyes are prerequisite to rapid silent reading."[2] When partial reading is used, the numerals of a problem are not read in detail and it has been shown that fewer pauses are required to get the meaning from the printed line. As a consequence, the average amount of material perceived at a pause is increased. It is clear, therefore, that both of the conditions which Freeman describes as representative of progress in reading are encouraged by the method of partial reading. At the same time, by employment of this method, the eye is relieved of the most severe exactions of the numerals and does not suffer the delay in movement which, owing to the nature of numerals as reading materials, is unavoidable when the numerals are read in detail. By virtue of this relief, the eye is able to approximate more nearly the rhythm of movement which is customary for lines of words, and which is "prerequisite to rapid silent reading."

4. APPLICATION OF RE-READING

The recommendations which appear above are concerned exclusively with the first reading of the problem. When the first reading is completed ordinarily the pupil is ready for the re-reading. It is possible, as was shown in chapter viii, to omit the work of re-reading by employing the method of mental computation while reading the problem as printed. The latter is a more economical procedure than the alternative method of computation from copied figures. So rapid a procedure as direct computation, however, is practicable only with easy problems and for pupils of unusual arithmetical ability. Probably the great majority of pupils solve the largest proportion of their problems with the aid of pencil and paper.

[1] F. N. Freeman, *op. cit.*, p. 83. [2] W. S. Gray, *op. cit.*, p. 39.

For all such pupils the work of re-reading and copying the numerals is an essential step in the process of solving. Extensive use of a method, however, does not guarantee successful accomplishment. By experience, and by careful investigation as well, teachers have learned that pupils do not copy numerals accurately. The situation, as it exists, is too serious to be neglected. Intelligent and vigorous efforts should be made to eliminate errors of this kind completely. Substantial improvement undoubtedly could be effected, by relieving the pupil's mind of every avoidable distraction during the copying. Setting apart a definite place for the step of copying, in the total procedure of solving, would constitute an important move in this direction. For this reason, the teacher's plan of instruction for arithmetical problems should include a special period of drill in re-reading and copying numerals.

When the plan of instruction has been amended, in accordance with this suggestion, it is important that the most efficient methods of reading the numerals be selected for use during the drill period. The methods of re-reading which were employed by the adult subjects of the present investigation are suggestive of the type of drill which should be given. For this reason the conclusions which were derived in chapter v were taken as the basis of recommendations concerning re-reading which are submitted as follows: (1) the numerals of any digit length should be read according to their dominant main-group pattern; and (2) the simplest possible numerical language should be used.

The effect of these recommendations and the advantages that lie in their adoption may be illustrated with the numerals 56,283 and 497. When the numeral 56,283 is treated as directed, it is read as follows: five six, two eight three. In this way the main-group pattern for five-digit numerals, which calls for two groups of two and three digits respectively, is followed. No words are used except such as are required to name the digits in order of succession. In the case of the numeral 497 the proper pronunciation is four, nine seven. In this instance the dominant main-group pattern for three-digit numerals is followed and no superfluous language is included.

In the reading of both numerals, advantage is taken of the strong natural tendency of the mind to arrange the members of a series of stimuli in groups. With the verbal description as brief as possible, no unnecessary waste of energy is incurred in pronunciation. In addition to the value of economy, an important reduction is effected in opportunity for errors by reading the numerals exactly as they are printed and precisely in the form in which they are to be copied. The feasi-

bility of the recommendations is beyond doubt since they are observed regularly in practice by various vocational groups, such as telephone operators and bookkeepers, who use numerals extensively in their daily work.

Nor are the directions as given applicable only to oral reading. It is a matter of common knowledge among students of reading that a very close connection exists between the inner speech of silent reading and the behavior of oral reading. Numerous elements of procedure are common to both. So far as the foregoing recommendations are concerned, there appears no necessity for drawing a distinction between oral and silent reading.

5. MISCELLANEOUS APPLICATIONS

Although the findings of the present investigation are not concerned extensively with the processes of computation, important data were presented in chapter viii concerning "computation direct from the problem card." The records show that this method is a very direct road to the answer and its use undoubtedly reduces the number of opportunities for error. The great rapidity with which the abler students can solve suitable problems in this manner would tend to increase the pupils' interest and concentration. For these reasons it is recommended that with abler students the process of direct computation be more extensively employed in rapid drill with problems.

The question will be raised as to the grade at which the methods of partial reading and re-reading should be introduced. The findings of the present investigation do not bear directly on this question. Nor can other than provisional conclusions be drawn until the reading of problems by children in the various grades has been studied. Significant inferences, however, can be drawn in the light of certain principles of method in teaching reading which were derived from scientific study and which are now generally accepted.

Partial reading and re-reading are methods of skilful and rapid silent reading. They represent a degree of achievement which is more advanced than mere ability to recognize the words. It is reasonably certain, therefore, that children are not prepared to read in this fashion before the teaching emphasis has been shifted from oral to silent reading, which, as is now generally known, should be done between the second and fourth grades. Beginning with the latter grade, progress in reading consists in large part in ability to master increasingly difficult materials. Arithmetical problems with several conditions and with longer numerals

constitute such materials and it is this type of problem which is attacked to advantage by the use of partial reading. In the nature of the case, partial reading and re-reading are highly specialized types of procedure. It is during the fourth, fifth, and sixth grades that pupils should be trained to use different types of reading ability. In view of this consideration, and of such others as are named above, it appears that the fourth grade is the appropriate time for the introduction of the new methods.

The discussion of applications ought not to be concluded without pointing out the fact that a body of experimental data in many instances is valuable for other purposes than those which were originally responsible for the investigation. Diagnosis of individual difficulties is an increasingly important feature of instruction in modern school systems. For such work, well-trained teachers and supervisors rely to as great an extent as is possible upon the materials which are available in scientific reports. No other body of material affords an equally detailed and illuminating description of the intellectual processes which are carried on in the ordinary work of the school. The description of first reading and re-reading and of certain computational processes, which is available in the present report, will prove useful for diagnosis of difficulties with arithmetical problems. Significant but less detailed descriptions of the range of correct recall of numerals, the grouping of digits, numerical-language patterns, effect of punctuation, and of such other items as a perusal of the table of contents will disclose, are also available for the same purpose.

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